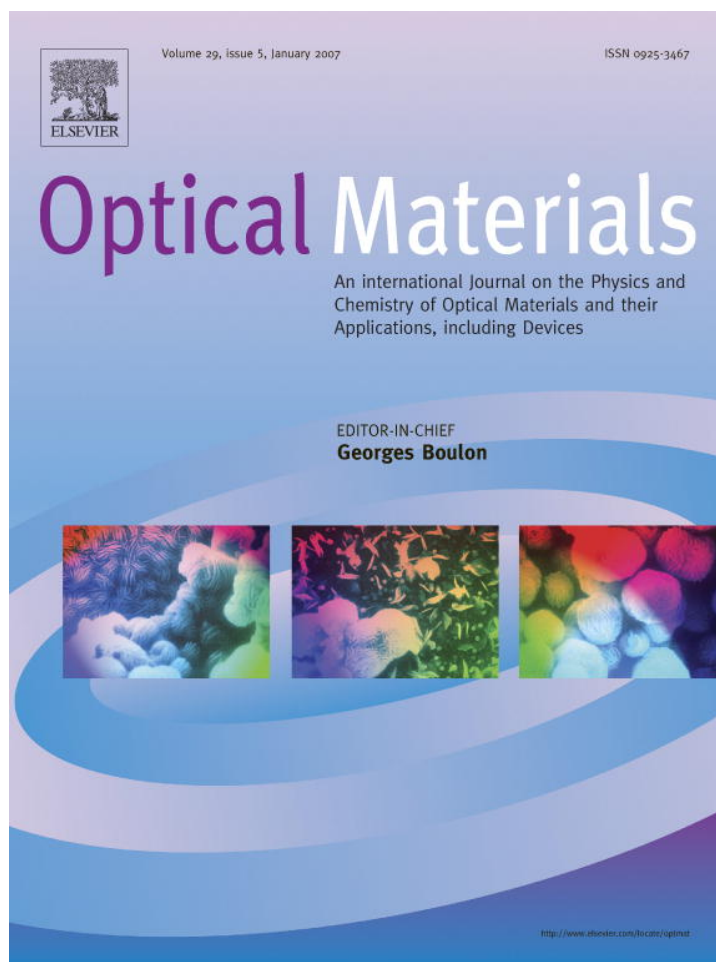


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The indicative surfaces of the photoelastic effect in Cs_2HgCl_4 biaxial crystals

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Received 2 June 2005; accepted 30 September 2005

Available online 9 January 2006

Abstract

We present a new method to calculate the indicative surfaces of the photoelastic effect in biaxial crystals. The corresponding equations are derived and applied to the calculations of the indicative surfaces of the longitudinal and transverse photoelastic effects in orthorhombic Cs_2HgCl_4 crystals. By means of the indicative surfaces the spatial anisotropy of photoelastic interaction in Cs_2HgCl_4 crystals is evaluated.

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PACS: 42.70.-a; 42.70.Nq; 42.79.Jq; 78.20.-e

Keywords: Indicative surface; Biaxial crystals; Photoelastic effect; Cs_2HgCl_4 crystals

1. Introduction

The last decade is characterized by a growing interest on the investigations of parametrical nonlinear optical phenomena (electro-optic, photoelastic or acousto-optic effects) in solid crystal and amorphous media. Concerning anisotropic materials the activity have been much shifted to low-symmetry crystals (see e.g., [1–6]), namely due to possibility of their applications in different optoelectronic devices, such as electro-optical and acousto-optical modulators and deflectors. The application of low-symmetry anisotropic materials in such devices (usually as media for electro-optical and acousto-optical cells) requires a knowledge on the spatial anisotropy for physical properties

of the crystals [1,7,8]. The main problems rise here in regard to the optimization of the electro-optic or photoelastic interaction geometry which only makes possible to use these materials with a maximal efficiency. Such problem may be solved in terms of the indicative surfaces (IS), which directly give a geometrical characterization of the spatial anisotropy of the parametrical optical effects represented by third- or fourth-rank tensors [9].

Present paper, deals with biaxial Cs_2HgCl_4 crystals. A Mach–Zehndner interferometer technique have been used to study the piezo-optical properties of Cs_2HgCl_4 crystals at room temperature giving a complete set of piezo-optical π_{mn} and photoelastic p_{in} tensor components [6,10]. Complete sets of the elastic tensor C_{ij} and the elastic modulus S_{ij} are known from the acoustical studies [11]. Substantial photoelastic effect and low ultrasonic velocities in these crystals determine relatively high figure of merit M_2 for the isotropic acousto-optical diffraction (for certain

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geometries of acousto-optical interactions M_2 is about of $100\text{--}110 \times 10^{-15} \text{ s}^3/\text{kg}$ [6]) thereby such material can be considered as rather promising candidate for applications in acousto-optical devices. This is the reason why the calculations of the IS of photoelastic effect in Cs_2HgCl_4 crystals are important, in particular for the optimization of the photoelastic interaction geometry giving further improvement of the acousto-optical efficiency.

2. The directional cosines for construction of the IS

In general case the components of the s -order rank tensor $T'_{unh\dots}$ in an arbitrary Cartesian coordinate system (where $u, n, h, \dots = 1, 2$ or 3 are the indices each corresponding to one of the orthogonal axes X'_1, X'_2 or X'_3) are related to the tensor components $T_{pcl\dots}$ in the principal crystallophysic system (where $p, c, l, \dots = 1, 2$ or 3 are the indices each corresponding to one of the principal axes X_1, X_2 or X_3) by the relation [9]:

$$T'_{unh\dots} = \alpha_{up}\alpha_{nc}\alpha_{hl}\dots T_{pcl\dots} \quad (1)$$

where $\alpha_{up}, \alpha_{nc}, \alpha_{hl}$ are the directional cosines of the rotated coordinate system X'_1, X'_2, X'_3 with respect to the principal coordinate system X_1, X_2, X_3 (see Fig. 1). In a case of the photoelastic effect these axes coincide with the directions of applied mechanical stress or light polarization. In order to build up the IS one must calculate tensor components T'_{unh} as the function of directional cosines $\alpha_{up}, \alpha_{nc}, \alpha_{hl}, \dots$. For the optical effects in uniaxial crystals the axes of the arbitrary rotated coordinate system are related with the direction of light propagation and its possible polarizations, what indeed decreases the number of IS [7,8]. We denote: \mathbf{k} is the direction of light propagation and \mathbf{r} is the radius vector of IS, which is parallel to one of the permitted light polarizations, \mathbf{j} coincides with the other light polarization component orthogonal to the vectors \mathbf{k} and \mathbf{r} . We choose $X'_1 \parallel \mathbf{k}, X'_2 \parallel \mathbf{j}$ and $X'_3 \parallel \mathbf{r}$ as is shown in Fig. 1 for uniaxial crystals. Thus the directional cosines yield a matrix of orthogonal transformation [8] from the principal crystallophysical coordinate system to the arbitrary rotated coordinate system. The directional cosines of the radius-vector \mathbf{r}

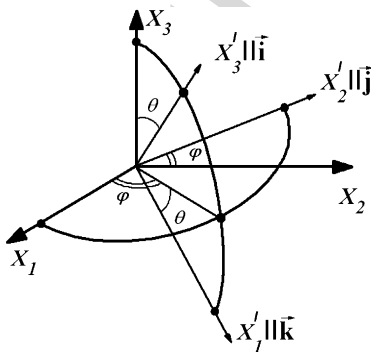


Fig. 1. Spatial position of the rotating coordinate system X'_1, X'_2, X'_3 with respect to the crystallophysical coordinate system X_1, X_2, X_3 for uniaxial crystals.

in spherical coordinates φ and θ can be written in the following well-known form:

$$\alpha_{r1} = \sin \theta \cos \varphi, \quad \alpha_{r2} = \sin \theta \sin \varphi, \quad \alpha_{r3} = \cos \theta \quad (2)$$

whereas the directional cosines for light polarization components \mathbf{k} and \mathbf{j} read as [8,9]:

$$\begin{aligned} \alpha_{k1} &= -\sin \varphi, \quad \alpha_{k2} = \cos \varphi, \quad \alpha_{k3} = 0, \\ \alpha_{j1} &= \cos \theta \cos \varphi, \quad \alpha_{j2} = \cos \theta \sin \varphi, \quad \alpha_{j3} = -\sin \theta \end{aligned} \quad (3)$$

where \mathbf{j} and \mathbf{r} coincide with the polarizations of the ordinary and extraordinary waves, respectively. By using these equations the indicative surfaces of longitudinal and transverse photoelastic effects were built up for uniaxial crystals of lithium niobate [7], $(\text{Ba}_x\text{Sr}_{1-x})\text{Nb}_2\text{O}_7$ [12] and β -barium borate [8]. This principle works for biaxial crystals too.

The dependence of two possible directions \mathbf{i} of light polarizations on a light propagation direction \mathbf{k} for biaxial crystals is deduced by Lagrange's undetermined multipliers λ, μ . A function F [13] is defined $F = 0.5\mathbf{i}\bar{\eta}\mathbf{i} - 0.5\lambda(\mathbf{i}\mathbf{i} - 1) + \mu(\mathbf{k}\mathbf{i})$, where $\bar{\eta}$ is impermeability tensor. Defining the extreme values in a plane perpendicular to the vector \mathbf{k} , one obtains [13]:

$$\partial F / \partial \mathbf{i} = \bar{\eta}\mathbf{i} - \lambda\mathbf{i} + \mu\mathbf{k} = \eta_{qq}\alpha_{iq} - \lambda\alpha_{iq} + \mu\alpha_{kq} = 0, \quad (4)$$

where $q = 1, 2, 3, \alpha_{iq}$ are the directional cosines corresponding to the vector \mathbf{i} , $\lambda = \mathbf{i}\bar{\eta}\mathbf{i} = \eta_{qq}\alpha_{iq}^2$ is the inverse squared refractive index of the light wave with the polarization parallel to the \mathbf{i} -direction and μ is a deflection of electric field vector of light wave from the wave front plane [13]. Eq. (4) can be also derived from the Maxwell's equations [13]. In crystallophysical coordinate system the tensor η has only diagonal components η_{qq} , so we will use its short notation η_q .

The directional cosines for \mathbf{k} and \mathbf{j} vectors can be defined by considering only one of light polarizations i.e., $\mathbf{i} \parallel \mathbf{r}$ in Eq. (4), thereby $\alpha_{iq} = \alpha_{rq}$. Expressing λ, α_{iq} , one obtains the directional cosines for the vector \mathbf{k} :

$$\begin{aligned} \alpha_{k1} &= \mu^{-1}(\Delta\eta_2 \cos^2 \theta - \Delta\eta_3 \sin^2 \theta \sin^2 \varphi) \sin \theta \cos \varphi, \\ \alpha_{k2} &= \mu^{-1}(\Delta\eta_3 \sin^2 \theta \cos^2 \varphi - \Delta\eta_1 \cos^2 \theta) \sin \theta \sin \varphi, \\ \alpha_{k3} &= \mu^{-1}(\Delta\eta_1 \sin^2 \theta \sin^2 \varphi - \Delta\eta_2 \cos^2 \theta) \sin^2 \theta \cos \theta, \end{aligned} \quad (5)$$

where $\Delta\eta_1 = \eta_2 - \eta_3$, $\Delta\eta_2 = \eta_3 - \eta_1$, $\Delta\eta_3 = \eta_1 - \eta_2$, which follow from the cyclic permutation rule of indices: 1–2–3, 3–1–2 or 2–3–1. In order to determine the directional cosines for \mathbf{j} , we take into account that $\mathbf{j} = [\mathbf{r}, \mathbf{k}]$, thus from Eqs. (5) and (2) we obtain:

$$\begin{aligned} \alpha_{j1} &= \Delta\eta_1 \mu^{-1} \sin \theta \cos \theta \sin \varphi, \\ \alpha_{j2} &= \Delta\eta_2 \mu^{-1} \sin \theta \cos \theta \cos \varphi, \\ \alpha_{j3} &= \Delta\eta_3 \mu^{-1} \sin^2 \theta \sin \varphi \cos \varphi. \end{aligned} \quad (6)$$

Since the sum of square of the directional cosines for \mathbf{k} and \mathbf{j} is equal one, from Eqs. (5) or (6) we get:

$$\begin{aligned} \mu = \pm \sin \theta \left((\Delta\eta_1)^2 \cos^2 \theta \sin^2 \varphi + (\Delta\eta_2)^2 \cos^2 \theta \cos^2 \varphi \right. \\ \left. + (\Delta\eta_3)^2 \sin^2 \theta \sin^2 \varphi \cos^2 \varphi \right)^{1/2}. \end{aligned} \quad (7)$$

To eliminate a sign ambiguity in Eq. (7) we consider the case $\theta = 0$ and $\varphi = 0$, i.e., when the rotated coordinate system coincides with crystallophysic one. In such case the directional cosines are defined as:

$$\alpha_{k1} = 1, \alpha_{k2} = 0, \alpha_{k3} = 0, \alpha_{j1} = 0, \alpha_{j2} = 1, \alpha_{j3} = 0, \quad (8)$$

thus by inserting Eq. (7) into Eqs. (5) and (6) the directional cosines for the vectors \mathbf{k} and \mathbf{j} get the following form:

$$\alpha_{k1} = \alpha_{j2} = \pm \Delta\eta_2 / \sqrt{(\Delta\eta_2)^2},$$

$$\alpha_{k2} = 0, \alpha_{k3} = 0, \alpha_{j1} = 0, \alpha_{j3} = 0. \quad (9)$$

For a coincidence of conditions (9) and (8), it is necessary to multiply Eqs. (5) and (6) on $\pm \sqrt{(\Delta\eta_2)^2} / \Delta\eta_2 = \pm |\Delta\eta_2| / \Delta\eta_2$. Then the sign of μ is well defined and the directional cosines for the vectors \mathbf{k} and \mathbf{j} have the form:

$$\alpha_{k1} = \mu' (\Delta\eta_2 \cos^2 \theta - \Delta\eta_3 \sin^2 \theta \sin^2 \varphi) \cos \varphi,$$

$$\alpha_{k2} = \mu' (\Delta\eta_3 \sin^2 \theta \cos^2 \varphi - \Delta\eta_1 \cos^2 \theta) \sin \varphi,$$

$$\alpha_{k3} = \mu' (\Delta\eta_1 \sin^2 \varphi - \Delta\eta_2 \cos^2 \varphi) \sin \theta \cos \theta,$$

$$\alpha_{j1} = \mu' \Delta\eta_1 \cos \theta \sin \varphi,$$

$$\alpha_{j2} = \mu' \Delta\eta_2 \cos \theta \cos \varphi,$$

$$\alpha_{j3} = \mu' \Delta\eta_3 \sin \theta \sin \varphi \cos \varphi, \quad (10)$$

where $\mu' = |\mu^{-1} \Delta\eta_2 \sin \theta| / \Delta\eta_2$. In the same way we have eliminated a sign ambiguity for the directional cosines of the vectors \mathbf{k} and \mathbf{j} . The relations (10) show a change of directional cosines as a function of angles θ and φ for \mathbf{k} and \mathbf{j} directions in biaxial crystals. They are general and thus can be also applied to particular cases, e.g., for uniaxial crystals ($\eta_1 = \eta_2$). The angles θ and φ define the direction of the radius-vector \mathbf{r} , which is parallel to one of the light polarization vectors \mathbf{i} . Using these relations, we can write the equations and construct the IS of photoelastic effect in biaxial crystals.

3. Construction and analysis of IS for the photoelastic effect in Cs₂HgCl₄ crystals

The general equation of the indicative surfaces for the longitudinal and transverse photoelastic effect in Cs₂HgCl₄ crystals directly follows from Eq. (1) by taking into account all nonzero components of the photoelastic tensor for the point group of symmetry *mmm*:

$$p'_{im} = \alpha_{i1}^2 \alpha_{m1}^2 p_{11} + \alpha_{i1}^2 \alpha_{m2}^2 p_{12} + \alpha_{i1}^2 \alpha_{m3}^2 p_{13} + \alpha_{i2}^2 \alpha_{m1}^2 p_{21}$$

$$+ \alpha_{i2}^2 \alpha_{m2}^2 p_{22} + \alpha_{i2}^2 \alpha_{m3}^2 p_{23} + \alpha_{i3}^2 \alpha_{m1}^2 p_{31} + \alpha_{i3}^2 \alpha_{m2}^2 p_{32}$$

$$+ \alpha_{i3}^2 \alpha_{m3}^2 p_{33} + 4\alpha_{i2} \alpha_{i3} \alpha_{m2} \alpha_{m3} p_{44}$$

$$+ 4\alpha_{i1} \alpha_{i3} \alpha_{m1} \alpha_{m3} p_{55} + 4\alpha_{i1} \alpha_{i2} \alpha_{m1} \alpha_{m2} p_{66}, \quad (11)$$

where α_{i1} , α_{i2} , α_{i3} and α_{m1} , α_{m2} , α_{m3} are the directional cosines of the polarization vector \mathbf{i} and the direction of applied mechanical deformation \mathbf{m} , respectively. In Eq. (11) all the components of the four-rank photoelastic tensor are given in the matrix representation (see e.g., [13,14]).

Inserting Eqs. (2) and (10) into Eq. (11), we obtain a set of three equations describing the IS of the longitudinal and transverse photoelastic effect, which read:

(1) IS of the longitudinal photoelastic effect $p'_{ii}(\theta, \varphi)$ $[\mathbf{i} \parallel \mathbf{m} \parallel \mathbf{r}]$:

$$p'_{ii}(\theta, \varphi) = \{p_{11} \cos^4 \varphi + (p_{12} + p_{21} + 4p_{66}) \sin^2 \varphi \cos^2 \varphi$$

$$+ p_{22} \sin^4 \varphi\} \sin^4 \theta + \{(p_{31} + p_{13} + 4p_{55}) \cos^2 \varphi$$

$$+ (p_{23} + p_{32} + 4p_{44}) \sin^2 \varphi\} \sin^2 \theta \cos^2 \theta + p_{33} \cos^4 \theta. \quad (12)$$

(2) IS of the transverse photoelastic effect for light polarization $p_{im}^{(i)}(\theta, \varphi) [\mathbf{i} \parallel \mathbf{r}, \mathbf{m} \parallel \mathbf{j}]$:

$$p_{im}^{(i)}(\theta, \varphi) = \left[\{p_{11} (\Delta\eta_1)^2 + p_{22} (\Delta\eta_2)^2 + p_{33} (\Delta\eta_3)^2$$

$$+ 4p_{44} \Delta\eta_3 \Delta\eta_2 + 4p_{55} \Delta\eta_3 \Delta\eta_1 + 4p_{66} \Delta\eta_2 \Delta\eta_1\} \sin^2 \theta$$

$$\times \cos^2 \theta \sin^2 \varphi \cos^2 \varphi + \{p_{12} (\Delta\eta_2)^2 \cos^4 \varphi$$

$$+ p_{21} (\Delta\eta_1)^2 \sin^4 \varphi\} \sin^2 \theta \cos^2 \theta + \{p_{13} (\Delta\eta_3)^2 \cos^2 \varphi$$

$$+ p_{23} (\Delta\eta_3)^2 \sin^2 \varphi\} \sin^4 \theta \sin^2 \varphi \cos^2 \varphi$$

$$+ \{p_{31} (\Delta\eta_1)^2 \sin^2 \varphi + p_{32} (\Delta\eta_2)^2 \cos^2 \varphi\} \cos^4 \theta \right]$$

$$\times \left[(\Delta\eta_1)^2 \cos^2 \theta \sin^2 \varphi + (\Delta\eta_2)^2 \cos^2 \theta \cos^2 \varphi$$

$$+ (\Delta\eta_3)^2 \sin^2 \theta \sin^2 \varphi \cos^2 \varphi \right]^{-1}. \quad (13)$$

(3) IS of the transverse photoelastic effect for mechanical deformation $p_{im}^{(m)}(\theta, \varphi) [\mathbf{m} \parallel \mathbf{r}, \mathbf{i} \parallel \mathbf{j}]$:

$$p_{im}^{(m)}(\theta, \varphi) = \left[\{p_{11} (\Delta\eta_1)^2 + p_{22} (\Delta\eta_2)^2 + p_{33} (\Delta\eta_3)^2$$

$$+ 4p_{44} \Delta\eta_3 \Delta\eta_2 + 4p_{55} \Delta\eta_3 \Delta\eta_1 + 4p_{66} \Delta\eta_2 \Delta\eta_1\} \sin^2 \theta$$

$$\times \cos^2 \theta \sin^2 \varphi \cos^2 \varphi + \{p_{21} (\Delta\eta_2)^2 \cos^4 \varphi$$

$$+ p_{12} (\Delta\eta_1)^2 \sin^4 \varphi\} \sin^2 \theta \cos^2 \theta + \{p_{31} (\Delta\eta_3)^2 \cos^2 \varphi$$

$$+ p_{32} (\Delta\eta_3)^2 \sin^2 \varphi\} \sin^4 \theta \sin^2 \varphi \cos^2 \varphi$$

$$+ \{p_{13} (\Delta\eta_1)^2 \sin^2 \varphi + p_{23} (\Delta\eta_2)^2 \cos^2 \varphi\} \cos^4 \theta \right]$$

$$\times \left[(\Delta\eta_1)^2 \cos^2 \theta \sin^2 \varphi + (\Delta\eta_2)^2 \cos^2 \theta \cos^2 \varphi$$

$$+ (\Delta\eta_3)^2 \sin^2 \theta \sin^2 \varphi \cos^2 \varphi \right]^{-1}, \quad (14)$$

where the upper index of $p_{im}^{(i)}$ or $p_{im}^{(m)}$ means which vector \mathbf{i} or \mathbf{m} is parallel to the radius-vector \mathbf{r} of the indicative surface. One must mention, that Eqs. (13) and (14) contain the value μ' , giving a positive sign ($\Delta\eta_2 = \eta_3 - \eta_1 > 0$) for the Cs₂HgCl₄ crystal. The indicative surfaces and their stereographic projections obtained by means of Eqs. (12)–(14) are shown in Fig. 2(a)–(c). In our calculations we inserted the magnitudes of the refractive indices $n_1 = 1.6498$,

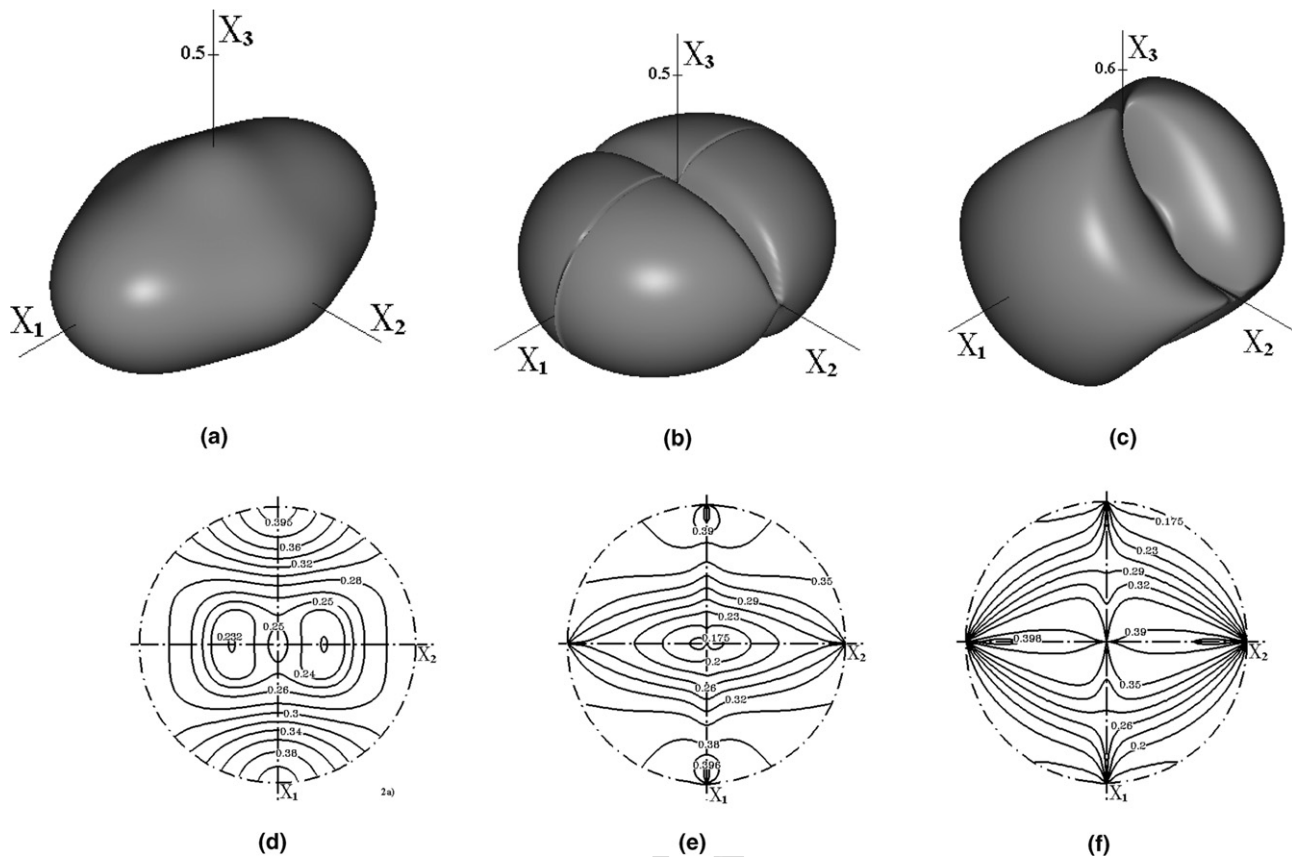


Fig. 2. The indicative surfaces (a)–(c) and their stereographic projections (d)–(f) of the longitudinal $p'_u(i)(\theta, \varphi)$ (a, d) and transverse $p'_im(i)(\theta, \varphi)$ (b, e), $p'_im(m)(\theta, \varphi)$ (c, f) photoelastic effects in Cs_2HgCl_4 crystals.

$n_2 = 1.669$ and $n_3 = 1.6491$ ($\lambda = 632.8$ nm) [6] and the magnitudes of photoelastic constants $p_{11} = 0.40$, $p_{12} = 0.40$, $p_{13} = 0.39$, $p_{21} = 0.26$, $p_{22} = 0.29$, $p_{23} = 0.34$, $p_{31} = 0.17$, $p_{32} = 0.19$, $p_{33} = 0.25$, $p_{44} = -0.034$, $p_{55} = -0.026$ and $p_{66} = -0.032$ [10].

General type construction of indicative surfaces was performed by means of new program Calc3D. This program is a ready product, working in Windows environment. The image is built up in spherical coordinate system by varying the angles θ and φ . In addition, it is possible to set a color and transparency for each point. Illumination normal to the surface were also taken into account. The invisible parts of surface were cut using Z-buffer algorithm. At developing of this program the technology of virtual machine and OpenGL library were used, that allows to obtain high quality images. The construction principle of stereographic projections is well-known and described in [9,13]. They are shown in Fig. 2(d)–(f).

The IS of the longitudinal and transverse photoelastic effects in Cs_2HgCl_4 crystals are characterized by the following properties:

- the IS in Fig. 2(a)–(c) do not possess a rotation symmetry in any direction that agree with German's theorem

[14]; however, they keep all the elements of mmm point symmetry, namely three mutually perpendicular two-fold axes and three planes of symmetry. This physical property is in agreement with the known principle of Curie–Neumann [9].

- the IS of the transverse effect (Fig. 2(b) and (c)) have a continuous set of values along the main axes. For instance, the effective magnitudes along X_1 , X_2 and X_3 axes are in the range of p_{12} – p_{13} , p_{21} – p_{23} , p_{31} – p_{32} for the IS $p'_im(i)(\theta, \varphi)$, and in the range of p_{21} – p_{31} , p_{12} – p_{32} , p_{13} – p_{23} for the IS $p'_im(m)(\theta, \varphi)$, respectively. Along the X_1 axis the change of the radius-vector of the IS $p'_im(i)(\theta, \varphi)$ from p_{12} to p_{13} is caused by a fixation of the direction $\mathbf{i} \parallel \mathbf{r} \parallel X_1$ with a possible change of the direction \mathbf{m} in the plane perpendicular to X_1 axis. In contrast to biaxial crystals, such continuous set of values is not observed along the main axes of uniaxial or cubic crystals, because $p_{31} = p_{32}$, $p_{13} = p_{23}$ and $\alpha_{j3} = 0$ in Eq. (3). Only the crystals of cubic symmetry 23 and $m3$ are exception. Their continuous set of values is observed along the X_3 -axis in accordance with peculiarity of their photoelastic tensor, i.e., due to the fact that $p_{31} \neq p_{32}$ and $p_{13} \neq p_{23}$ (see e.g., [15]).
- the analytical search for extreme values at IS in the case of Cs_2HgCl_4 crystals, as it was done for lithium niobate

[7] or β -barium borate [8] crystals by applying the conditions $\partial p/\partial\theta = 0$ and $\partial p/\partial\varphi = 0$, can be obtained only for the longitudinal effect. So Eq. (12) to solve is of the type:

$$p'_{ii}(\varphi, \theta) = a(\varphi) \sin^4(\theta) + b(\varphi) \sin^2(\theta) \cos^2(\theta) + c \cos^4(\theta). \quad (15)$$

Differentiating with respect to θ yields

$$\frac{\partial}{\partial\theta} p'_{ii}(\varphi, \theta) = 2 \sin\theta \cos\theta (2[a(\varphi) - b(\varphi) + c] \sin^2\theta + [b(\varphi) - 2c]). \quad (16)$$

Therefore, extreme values occur for $\sin\theta = 0$, $\cos\theta = 0$ and $(2[a(\varphi) - b(\varphi) + c]\sin^2\theta + [b(\varphi) - 2c]) = 0$. Thus it is evident, that the extremal values occur at $\theta_{\text{extr.}} = 0, \pi/2, \pi, \dots$ and

$$\tilde{\theta}_{\text{extr.}} = \arcsin \left\{ \sqrt{\frac{1}{2} \frac{b(\varphi) - 2c}{a(\varphi) - b(\varphi) + c}} \right\}. \quad (17)$$

According to (17) we can calculate the $\tilde{\theta}_{\text{extr.}}$ for different values of ϕ . For $\phi = 90^\circ$ one obtains $\tilde{\theta}_{\text{extr.}}$:

$$\tilde{\theta}_{\text{extr.}} = \arcsin \left\{ \sqrt{\frac{1}{2} \frac{p_{23} + p_{32} + 4p_{44} - 2p_{33}}{p_{22} - (p_{23} + p_{32} + 4p_{44}) + p_{33}}} \right\}, \quad (18)$$

which in the case of Cs_2HgCl_4 crystals is equal to 37° . Inserting this, the magnitude of extremal longitudinal effect amounts to

$$\begin{aligned} \lim_{\varphi \rightarrow \pi/2} \left(\lim_{\theta \rightarrow \tilde{\theta}_{\text{extr.}}} [p'_{ii}(\theta, \varphi)] \right) &= \frac{1}{4} \frac{(p_{23} + p_{32} + 4p_{44})^2 - 4p_{22}p_{33}}{(p_{23} + p_{32} + 4p_{44}) - (p_{22} + p_{33})} \\ &= \tilde{p} \text{ (longitudinal minimum)}, \end{aligned} \quad (19)$$

which in our case is equal to 0.23. This information might be useful, as it covers the dependence on the photoelastic components. Besides such extreme values also the extreme values for transverse effect were obtained numerically using our software and are presented in Table 1 (here the angular parameters of directions are given only for radius-vector of IS). Knowing the extreme directions for the longitudinal and transverse effects one may take the limits to obtain corresponding extremal magnitudes:

$$\begin{aligned} \lim_{\varphi \rightarrow 0} \left(\lim_{\theta \rightarrow \pi/2} [p'_{ii}(\theta, \varphi)] \right) &= p_{11} \text{ (longitudinal maximum)}, \\ \lim_{\varphi \rightarrow \pi/2} \left(\lim_{\theta \rightarrow 0} [p_{im}^{(i)}(\theta, \varphi)] \right) &= p_{31} \text{ (transverse minimum)}, \\ \lim_{\theta \rightarrow \pi/2} \left(\lim_{\varphi \rightarrow 0} [p_{im}^{(i)}(\theta, \varphi)] \right) &= p_{12} \text{ (transverse maximum)}, \\ \lim_{\varphi \rightarrow 0} \left(\lim_{\theta \rightarrow \pi/2} [p_{im}^{(m)}(\theta, \varphi)] \right) &= p_{31} \text{ (transverse minimum)}, \\ \lim_{\theta \rightarrow \pi/2} \left(\lim_{\varphi \rightarrow \pi/2} [p_{im}^{(m)}(\theta, \varphi)] \right) &= p_{12} \text{ (transverse maximum)}. \end{aligned} \quad (20)$$

One must be stressed that the order of taking the limits is important here, i.e., $\lim_{\varphi}(\lim_{\theta}) \neq \lim_{\theta}(\lim_{\varphi})$. Table 1 presents the angular parameters for only one direction neglecting all other symmetry equivalent directions. The latter ones may be obtained by employing a reflecting of the corresponding radius vectors in three crystallographic planes of symmetry. All the IS are characterized by almost the same magnitudes in their extreme points because the values of p_{11} and p_{12} constants are nearly equal for Cs_2HgCl_4 crystals.

- the anisotropy power for all IS is also shown in Table 1. It was obtained using the relation [8]: $\gamma = (V_{\text{sphere}} - |V^+ - V^-|)100\%/V_{\text{sphere}}$, where $V_{\text{sphere}} = 4\pi|f_{\text{extr.}}|^3/3$ is the sphere volume corresponding to a certain radius: $|f_{\text{extr.}}| = \max(|f_{\text{min}}|, |f_{\text{max}}|)$; f_{min} and f_{max} are the minimum and maximum values of the IS, respectively, V^+ and V^- are the volumes of the positive and negative parts of the surface, respectively. Taking into account, that all IS in Fig. 2(a)–(c) have the positive parts only, we have applied a simplified formula for the calculation of anisotropy power, i.e., $\gamma = (V_{\text{sphere}} - V^+)100\%/V_{\text{sphere}}$.
- comparing the IS of photoelastic effect (see Fig. 2(a)–(c)) and of the piezo-optical one [16], one may realize a substantial difference, in particular piezo-optical effect shows a more complicated structure of IS, including the inversion of the sign for both transverse IS $\pi_{im}^{(i)}(\theta, \varphi)$ and $\pi_{im}^{(m)}(\theta, \varphi)$. That is why the anisotropy power for piezo-optical effect [16] is about of 1.5–2 times larger as compared to the photoelastic effect. The directions corresponding to the extreme values of transverse piezo- and photoelastic effects practically coincide whereas for the longitudinal effect [$\pi'_{ii}(\theta, \varphi)$ and $p'_{ii}(\theta, \varphi)$] they clearly differ. It may be explained by the fact that $p_{33} > p_{11}$ for the piezo-optical effect and $p_{33} < p_{11}$ for photoelastic effect.

Table 1
The extreme values and anisotropy power for IS of photoelastic effect in Cs_2HgCl_4 crystals

| IS | Minimal magnitude for the IS and its direction | | | Maximum magnitude for the IS and its direction | | | Anisotropy power | | |
|---------------------------------|--|--------------|---------------|--|--------------|---------------|---|---------------------------|--------------|
| | Magnitude | θ (°) | φ (°) | Magnitude | θ (°) | φ (°) | V_{sphere} (unit) ³ | V^+ (unit) ³ | γ (%) |
| $p'_{ii}(\theta, \varphi)$ | $\tilde{p} = 0.23$ | 37 | 90 | $p_{11} = 0.40$ | 90 | 0 | 0.80 | 0.32 | 60 |
| $p_{im}^{(i)}(\theta, \varphi)$ | $p_{31} = 0.17$ | 0 | 0 | $p_{12} = 0.40$ | 90 | 0 | 0.80 | 0.36 | 55 |
| $p_{im}^{(m)}(\theta, \varphi)$ | $p_{31} = 0.17$ | 90 | 0 | $p_{12} = 0.40$ | 90 | 90 | 0.80 | 0.42 | 48 |

Therefore the extreme value for longitudinal IS $\pi'_{ii}(\theta, \varphi)$ occurs at $\theta = 0^\circ$, i.e. along the X_3 axis whereas for IS $p'_{ii}(\theta, \varphi)$ the extreme magnitude is at $\theta = 90^\circ$ and $\varphi = 0^\circ$, i.e., along the X_1 axis.

4. Conclusions

In conclusion we present here a new method to calculate the IS for the photoelastic effect in biaxial crystals. The corresponding equations has been derived and applied to the calculations of the IS related to the longitudinal and transverse photoelastic effects in orthorhombic Cs_2HgCl_4 crystals. The maximum magnitudes at the IS of the longitudinal and transverse photoelastic effects are revealed along the principal crystallographic directions. This indeed defines a set of the optimized sample geometries recommended for designers that develop acousto-optical modulators and deflectors built up using Cs_2HgCl_4 crystals as photoelastic media. By means of the IS the spatial anisotropy of the photoelastic interaction in Cs_2HgCl_4 crystals is evaluated.

Acknowledgement

Present work was supported by Science and Technology Center in Ukraine (project no. #3222).

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