

Two-fold interferometric measurements of piezo-optic constants: application to β -BaB₂O₄ crystals

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Abstract

We present the interferometric technique which allows to measure piezooptical and photoelastic characteristics of crystal materials of any symmetry. The offered two-fold interferometric method enables to determine all independent non-zero piezooptic and photoelastic constants by measuring pressure induced changes of optical path. As advantage to known acoustooptical techniques this method allows to measure both the absolute magnitude and sign of photoelastic constants. In general case the determination of 36 components of piezooptic tensor needs to carry out 57 measurements on 16 samples. The corresponding relationships are derived. As an example we apply here the interferometric technique to measure the piezooptic and photoelastic constants in trigonal β -BaB₂O₄ crystals.

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1. Introduction

Photoelasticity is a well-known phenomenon that was observed for the first time by Brewster two centuries ago [1]. At present, this effect has considerable practical importance and is applicable in such areas of optoelectronics as acoustooptical devices, piezooptic sensors etc. The application of the photoelastic effect is usually related with two problems. The first one is a search for new materials with high magnitudes of the photoelastic constants. The second one is a choice of optimal sample geometry which is characterized by the most effective photoelastic transformation. These problems are actually related with a spatial distribution analysis of the photoelastic effect [2,3] which requires to know both values and signs of all independent nonzero components of the fourth-rank piezooptic or photoelastic tensors. It

should be noted that acoustooptical methods are unable to determine the sign of photoelastic constants, especially in the case of low-symmetry crystals [4]. Such ambiguity leads to sufficient errors in further calculations, namely in the case when the determination of all tensor components is required.

In this paper we present the interferometric technique which allows to measure the piezooptic and photoelastic characteristics of crystal materials of any symmetry. The two-fold interferometric method offered here enables to determine all independent non-zero piezooptic and photoelastic constants by measuring piezo-induced changes of optical path. As an advantage to acoustooptical techniques this method also allows to measure both the absolute magnitude and sign of photoelastic constants. The purpose of this article is thereby to give the description of measuring procedures as well as to present the derived relationships that will be necessary for corresponding calculations. As an example we apply here the interferometric technique to measure the

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piezooptic and photoelastic constants of trigonal β -BaB₂O₄ crystals.

2. The relationships for determination of the piezooptic constants

Both the sign and the absolute magnitude of piezooptic constants can be determined in an unambiguous way by satisfying the following requirements:

1. For any crystal material which is characterized by a certain point group of symmetry, one has to choose an appropriate axes set regarding to a crystallophysic coordinate system (for details see e.g. Refs. [5,6]).
2. One must measure the changes of optical path $\delta\Delta_{ikm}$ that occur under an applied mechanical stress σ_m . The sign of the piezo-induced changes is determined according to the following criterion: if applied stress σ_m increases the optical path of the beam, then $\delta\Delta_{ikm}$ is positive, otherwise $\delta\Delta_{ikm}$ is negative. The mechanical compression is considered to be always negative.

The piezooptic constants are usually measured by the interferometric methods described in detail in Ref. [4]. The measurements of piezooptic effect were carried out by using Mach–Zehnder interferometer and the experimental setup is shown in Fig. 1. The semitransparent mirror 2 splits the incident laser radiation on the principal (I) and the reference (II) beams. The resulting interferometric pattern is projected by the lens 10 on the gap 11, the light intensity is registered by the photodetector 12 connected with measuring electronic circuit (amplifier + A/D-converter). The polarizer 7 sets the orientation of incident polarization. The analyzer 9 is applied only in the cases when the piezooptic constants of an induced birefringence (i.e. the effective piezooptic constant π_{km}^*) are measured. Several additional steps have been done in order to reduce an influence of random vibrations and uncontrolled temperature variations on the measurement precision. In

particular, all the mentioned optical elements have been mounted on the optical table equipping by a pneumatic vibration isolation system. The measurement have been performed in a closed acclimatization room supporting nearly a constant room temperature $20 \pm 0.25^\circ\text{C}$. Laser interferometers have one substantial advantage: due to a large coherence length of the lasers (about 0.2 m), there is no need any more in routines related with an adjusting of optical paths concerning the principal and reference channels of the interferometer; as a result quite pronounced interferometric patterns can be easily obtained even without the identical sample which had to be put into the reference beam when one deals with low coherent light sources.

The relation between the induced changes of optical path $\delta\Delta'_{ikm}$ and applied mechanical stress σ_m can be presented as follows:

$$\delta\Delta'_{ikm} = (-\pi'_{im}n_i^3/2 + S'_{km}(n_i - 1))\sigma_m t_k, \tag{1}$$

where S'_{km} are the elastic compliances, n'_i are the refractive indices, π'_{im} are the piezooptic constants, t_k is the length of the sample in the direction of light propagation (sample is placed in one of interferometer arms). In fact Eq. (1) can be simply obtained by differentiation of the optical path $\Delta'_{ikm} = (n'_i - n_c)t_k$ with respect to σ_m , where n_c is the refractive index of medium, in which interferometer is accommodated (usually this is the air, for which $n_c = 1$). Here π'_{im} , S'_{km} and n'_i are the constants for a certain arbitrary chosen coordinate system. They are related to the constants π_{fghq} , S_{fghq} , and n_i of the principal crystallophysic coordinate system as:

$$\pi'_{im} = \pi'_{iimm} = \alpha_{if}\alpha_{ig}\alpha_{mh}\alpha_{mq}\pi_{fghq}, \tag{2}$$

$$S'_{km} = S'_{kkmm} = \alpha_{kf}\alpha_{kq}\alpha_{mh}\alpha_{mq}S_{fghq}, \tag{3}$$

$$n'_i = (\alpha_{ij}^2 n_i^{-2})^{-1/2}, \tag{4}$$

where α_{ij} are the directional cosines defining the geometry of photoelastic interaction, i.e. the directions

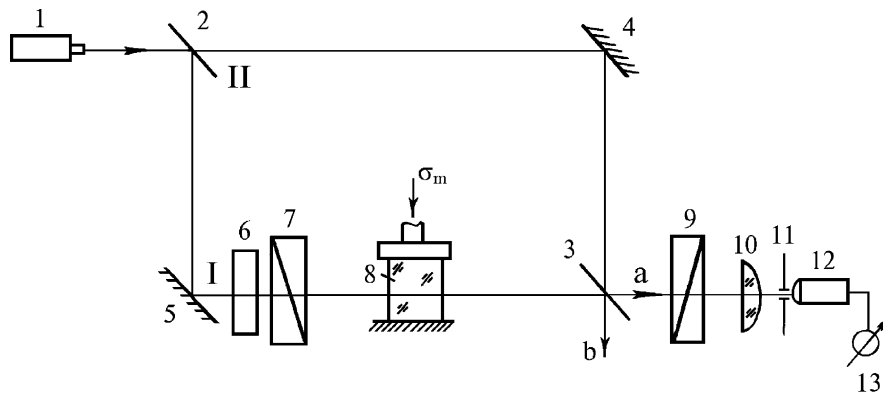


Fig. 1. Experimental setup of two-fold measurements method based on Mach-Zehnder interferometer: 1 is the He-Ne laser, 2 and 3 are the semitransparent mirrors, 4 and 5 are the mirrors, 6 is the $\pi/4$ -plate, 7 is the polarizer, 8 is the sample; 9 is the analyzer; 10 is the lens, 11 is the gap; 12 is the photodetector; 13 is the registration electronic circuit.

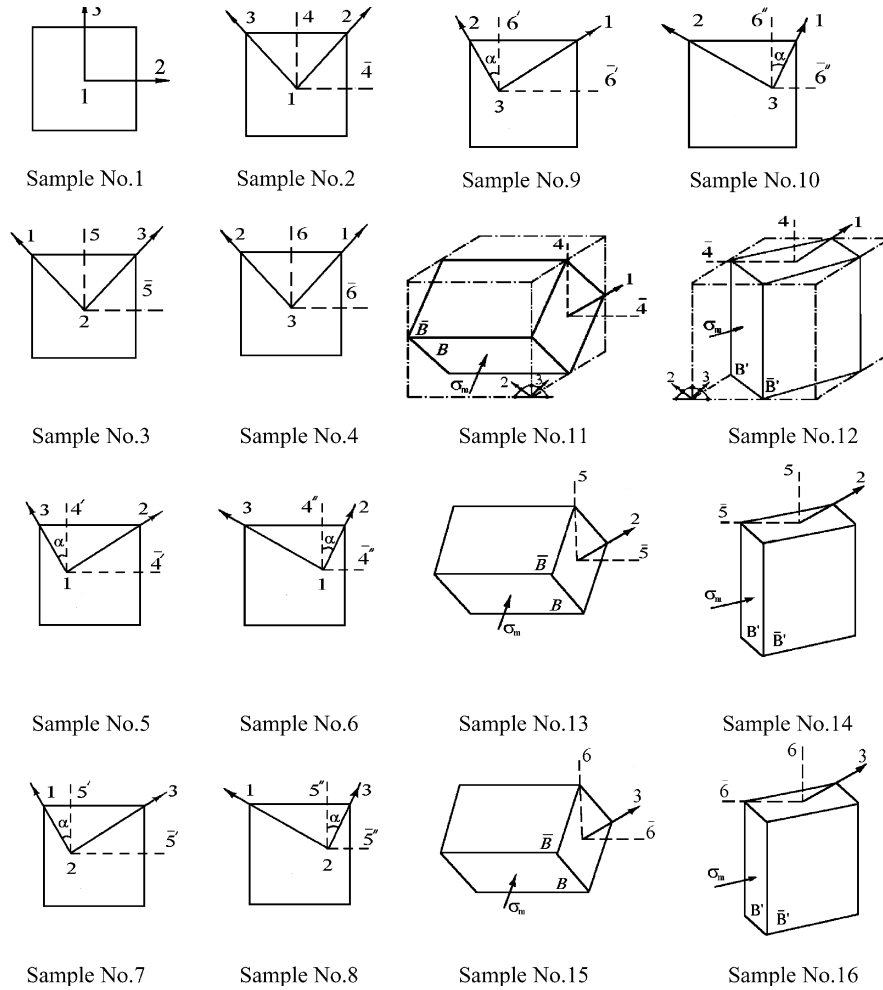


Fig. 2. The set of the samples that are necessary to measure in order to determine all the components of piezooptic (photoelastic) tensor in crystals of triclinic symmetry.

of application of normal mechanical stress \mathbf{m} , light polarization \mathbf{i} and light propagation \mathbf{k} , respectively. By combining (1)–(4) we can derive the equations which represents the induced changes of the optical path $\delta\Delta'_{ikm}$ in a sample as a function of linear combination of absolute piezooptic constants π_{im} , elastic compliances S_{km} and the principal refractive indices n_i ($i = 1, 2, 3$). By choosing suitable geometries of photoelastic interaction, one can consistently (even for triclinic crystals) determine π_{im} from experimentally measured $\delta\Delta'_{ikm}$. Let us illustrate this on several typical cases.

1. The piezooptic constants π_{im} ($i, m = 1, 2, 3$) can be determined by performing the measurements on one direct-cut sample (see Fig. 2, sample no. 1). In this case one may use the simplified version of Eq. (1) (Table 1, formula (T.1)), which can be obtained by inserting all necessary directional cosines concerning the unit vectors $\mathbf{i}, \mathbf{k}, \mathbf{m}$ into (2)–(4).

2. The piezooptic constants π_{im} ($i = 1, 2, 3, m = 4, 5, 6$). As a typical example, let us consider in detail the calculation procedure for π_{14} -component only. In this case one have to use the X/45°-cut sample (Fig. 2,

sample no. 2), for which the measurements can be performed in two different experimental conditions depending on a sample set: $i = 1, k = \bar{4}, m = 4$ [called as direct conditions, here 4 is the diagonal direction between positive directions of X_2 and X_3 axis, the direction $\bar{4}$ is the diagonal direction between negative directions of X_2 (or X_3) axis and positive direction of the X_3 (or X_2) axis] and $i = 1, k = 4, m = \bar{4}$ (called as symmetric conditions). For both these conditions, the directional cosines can be written as

$$\begin{aligned} \alpha_{i1} &= 1, & \alpha_{k1} &= 0, & \alpha_{m1} &= 0, \\ \alpha_{i2} &= 0, & \alpha_{k2} &= \sqrt{2}/2, & \alpha_{m2} &= \sqrt{2}/2, \\ \alpha_{i3} &= 0, & \alpha_{k3} &= \mp\sqrt{2}/2, & \alpha_{m3} &= \pm\sqrt{2}/2. \end{aligned} \tag{5}$$

Here and in all equations below the lower sign corresponds to the case of symmetric conditions. Inserting Eq. (5) into Eqs. (2)–(4) we obtain

$$\begin{aligned} \pi'_{im} &= (\pi_{12} + \pi_{13} \pm \pi_{14})/2, \\ S'_{km} &= (S_{22} + S_{33} + 2S_{23} - S_{44})/2, & n'_i &= n_i. \end{aligned} \tag{6}$$

Table 1
The relationships for calculations of piezooptic constants in two-fold interferometric method derived for the crystals of triclinic symmetry

Necessary samples (see Fig. 1)	Relationships ^b	Calculated error ^a	No. of equation
No. 1	$\pi_{im} = -2n_i^{-3} \frac{\partial \Delta_{ikm}}{\partial \sigma_m} + 2n_i^{-3} (n_i - 1) S_{km}$, where $i, k, m = 1, 2, 3$	~5% (~5%)	T.1
No. 2	$\pi_{14} = -2n_1^{-3} \left(\frac{\partial \Delta_{144}}{\partial \sigma_4} - \frac{\partial \Delta_{144}}{\partial \sigma_4} \right)$	~5% (~8%)	T.2
No. 3	$\pi_{25} = -2n_2^{-3} \left(\frac{\partial \Delta_{255}}{\partial \sigma_5} - \frac{\partial \Delta_{255}}{\partial \sigma_5} \right)$	~5% (~8%)	T.3
No. 4	$\pi_{36} = -2n_3^{-3} \left(\frac{\partial \Delta_{366}}{\partial \sigma_6} - \frac{\partial \Delta_{366}}{\partial \sigma_6} \right)$	~5% (~8%)	T.4
No. 2	$2\pi_{41} = -2n_4^{-3} \left(\frac{\partial \Delta_{441}}{\partial \sigma_1} - \frac{\partial \Delta_{441}}{\partial \sigma_1} \right) - 2n_4^{-3} (n_4 - 1) S_{14}$	~5% (~8%)	T.5
No. 3	$2\pi_{52} = -2n_5^{-3} \left(\frac{\partial \Delta_{552}}{\partial \sigma_2} - \frac{\partial \Delta_{552}}{\partial \sigma_2} \right) - 2n_5^{-3} (n_5 - 1) S_{25}$	~5% (~8%)	T.6
No. 4	$2\pi_{63} = -2n_6^{-3} \left(\frac{\partial \Delta_{663}}{\partial \sigma_3} - \frac{\partial \Delta_{663}}{\partial \sigma_3} \right) - 2n_6^{-3} (n_6 - 1) S_{36}$	~5% (~8%)	T.7
No. 2	$\pi_{24} = -2n_2^{-3} \left(\frac{\partial \Delta_{214}}{\partial \sigma_4} - \frac{\partial \Delta_{214}}{\partial \sigma_4} \right) + 2n_2^{-3} (n_2 - 1) S_{14}$	~5% (~8%)	T.8
No. 2	$\pi_{34} = -2n_3^{-3} \left(\frac{\partial \Delta_{314}}{\partial \sigma_4} - \frac{\partial \Delta_{314}}{\partial \sigma_4} \right) + 2n_3^{-3} (n_3 - 1) S_{14}$	~5% (~8%)	T.9
No. 3	$\pi_{15} = -2n_1^{-3} \left(\frac{\partial \Delta_{125}}{\partial \sigma_5} - \frac{\partial \Delta_{125}}{\partial \sigma_5} \right) + 2n_1^{-3} (n_1 - 1) S_{25}$	~5% (~8%)	T.10
No. 3	$\pi_{35} = -2n_3^{-3} \left(\frac{\partial \Delta_{325}}{\partial \sigma_5} - \frac{\partial \Delta_{325}}{\partial \sigma_5} \right) + 2n_3^{-3} (n_3 - 1) S_{25}$	~5% (~8%)	T.11
No. 4	$\pi_{16} = -2n_1^{-3} \left(\frac{\partial \Delta_{136}}{\partial \sigma_6} - \frac{\partial \Delta_{136}}{\partial \sigma_6} \right) + 2n_1^{-3} (n_1 - 1) S_{36}$	~5% (~8%)	T.12
No. 4	$\pi_{26} = -2n_2^{-3} \left(\frac{\partial \Delta_{236}}{\partial \sigma_6} - \frac{\partial \Delta_{236}}{\partial \sigma_6} \right) + 2n_2^{-3} (n_2 - 1) S_{36}$	~5% (~8%)	T.13
No. 2	$\pi_{44} = -2n_4^{-3} \left(\frac{\partial \Delta_{444}}{\partial \sigma_4} + \frac{\partial \Delta_{444}}{\partial \sigma_4} \right) + n_4^{-3} (n_4 - 1) (S_{22} + S_{33} + 2S_{23} - S_{44}) - (\pi_{22} + \pi_{23} + \pi_{32} + \pi_{33})/2$	~11% (~11%)	T.14
No. 3	$\pi_{55} = -2n_5^{-3} \left(\frac{\partial \Delta_{555}}{\partial \sigma_5} + \frac{\partial \Delta_{555}}{\partial \sigma_5} \right) + n_5^{-3} (n_5 - 1) (S_{11} + S_{33} + 2S_{13} - S_{55}) - (\pi_{11} + \pi_{13} + \pi_{31} + \pi_{33})/2$	~11% (~11%)	T.15
No. 4	$\pi_{66} = -2n_6^{-3} \left(\frac{\partial \Delta_{666}}{\partial \sigma_6} + \frac{\partial \Delta_{666}}{\partial \sigma_6} \right) + n_6^{-3} (n_6 - 1) (S_{11} + S_{22} + 2S_{12} - S_{66}) - (\pi_{11} + \pi_{12} + \pi_{21} + \pi_{22})/2$	~11% (~11%)	T.16
No. 2, 5, 6	$\pi_{42} = -3n_4^{-3} \left(\frac{\partial \Delta_{444}}{\partial \sigma_4} - \frac{\partial \Delta_{444}}{\partial \sigma_4} \right) + \frac{4\sqrt{3}}{3} n_4^{-3} \left(\frac{\partial \Delta_{4'4'4'}}{\partial \sigma_{4'}} - \frac{\partial \Delta_{4''4''4''}}{\partial \sigma_{4''}} \right) + n_4^{-3} (n_4 - 1) (S_{34} - S_{24}) - \pi_{24}/2$	~9% (~19%) ~9%	T.17 T.18

Table 1 (continued)

Necessary samples (see Fig. 1)	Relationships ^b	Calculated error ^a	No. of equation
No. 3, 7, 8	$\pi_{43} = n_4^{-3} \left(\frac{\delta\Delta_{444}}{i_4 \cdot \sigma_4} - \frac{\delta\Delta_{444}}{i_4 \cdot \sigma_4} \right) - \frac{4\sqrt{3}}{3} n_4^{-3} \left(\frac{\delta\Delta_{4'4'4'}}{i_4' \cdot \sigma_4'} - \frac{\delta\Delta_{4''4''4''}}{i_4'' \cdot \sigma_4''} \right) - n_4^{-3} (n_4 - 1)(S_{34} - S_{24}) - \pi_{34}/2$	(~ 19%)	T.19
	$\pi_{53} = -3n_5^{-3} \left(\frac{\delta\Delta_{555}}{i_5 \cdot \sigma_5} - \frac{\delta\Delta_{555}}{i_5 \cdot \sigma_5} \right) + \frac{4\sqrt{3}}{3} n_5^{-3} \left(\frac{\delta\Delta_{5'5'5'}}{i_5' \cdot \sigma_5'} - \frac{\delta\Delta_{5''5''5''}}{i_5'' \cdot \sigma_5''} \right) + n_5^{-3} (n_5 - 1)(S_{15} - S_{35}) - \pi_{35}/2$	~ 9%	
	$\pi_{51} = n_5^{-3} \left(\frac{\delta\Delta_{555}}{i_5 \cdot \sigma_5} - \frac{\delta\Delta_{555}}{i_5 \cdot \sigma_5} \right) - \frac{4\sqrt{3}}{3} n_5^{-3} \left(\frac{\delta\Delta_{5'5'5'}}{i_5' \cdot \sigma_5'} - \frac{\delta\Delta_{5''5''5''}}{i_5'' \cdot \sigma_5''} \right) - n_5^{-3} (n_5 - 1)(S_{15} - S_{35}) - \pi_{15}/2$	(~ 19%) ~ 9%	
No. 4, 9, 10	$\pi_{61} = -3n_6^{-3} \left(\frac{\delta\Delta_{666}}{i_6 \cdot \sigma_6} - \frac{\delta\Delta_{666}}{i_6 \cdot \sigma_6} \right) + \frac{4\sqrt{3}}{3} n_6^{-3} \left(\frac{\delta\Delta_{6'6'6'}}{i_6' \cdot \sigma_6'} - \frac{\delta\Delta_{6''6''6''}}{i_6'' \cdot \sigma_6''} \right) + n_6^{-3} (n_6 - 1)(S_{26} - S_{16}) - \pi_{16}/2$	(~ 19%) ~ 9%	T.21
	$\pi_{62} = n_6^{-3} \left(\frac{\delta\Delta_{666}}{i_6 \cdot \sigma_6} - \frac{\delta\Delta_{666}}{i_6 \cdot \sigma_6} \right) - \frac{4\sqrt{3}}{3} n_6^{-3} \left(\frac{\delta\Delta_{6'6'6'}}{i_6' \cdot \sigma_6'} - \frac{\delta\Delta_{6''6''6''}}{i_6'' \cdot \sigma_6''} \right) - n_6^{-3} (n_6 - 1)(S_{26} - S_{16}) - \pi_{26}/2$	(~ 19%) ~ 9%	
	$\pi_{63} = n_6^{-3} \left(\frac{\delta\Delta_{666}}{i_6 \cdot \sigma_6} - \frac{\delta\Delta_{666}}{i_6 \cdot \sigma_6} \right) - \frac{4\sqrt{3}}{3} n_6^{-3} \left(\frac{\delta\Delta_{6'6'6'}}{i_6' \cdot \sigma_6'} - \frac{\delta\Delta_{6''6''6''}}{i_6'' \cdot \sigma_6''} \right) - n_6^{-3} (n_6 - 1)(S_{26} - S_{16}) - \pi_{26}/2$	(~ 19%) ~ 9%	
No. 11, 12	$\pi_{46} = \sqrt{2} n_4^{-3} \left\{ \left(\frac{\delta\Delta_{44B}}{i_4 \cdot \sigma_B} - \frac{\delta\Delta_{44B}}{i_4 \cdot \sigma_B} \right) + \left(\frac{\delta\Delta_{44B'}}{i_4 \cdot \sigma_{B'}} - \frac{\delta\Delta_{44B'}}{i_4 \cdot \sigma_{B'}} \right) \right\} + n_4^{-3} (n_4 - 1)(S_{25} + S_{35} - S_{46}) - (\pi_{25} + \pi_{35})/2$	(~ 19%) ~ 14%	T.23
	$\pi_{45} = \sqrt{2} n_4^{-3} \left\{ \left(\frac{\delta\Delta_{44B}}{i_4 \cdot \sigma_B} - \frac{\delta\Delta_{44B}}{i_4 \cdot \sigma_B} \right) - \left(\frac{\delta\Delta_{44B'}}{i_4 \cdot \sigma_{B'}} - \frac{\delta\Delta_{44B'}}{i_4 \cdot \sigma_{B'}} \right) \right\} + n_4^{-3} (n_4 - 1)(S_{26} + S_{36} - S_{45}) - (\pi_{26} + \pi_{36})/2$	(~ 32%) ~ 14%	
	$\pi_{45} = \sqrt{2} n_4^{-3} \left\{ \left(\frac{\delta\Delta_{44B}}{i_4 \cdot \sigma_B} - \frac{\delta\Delta_{44B}}{i_4 \cdot \sigma_B} \right) - \left(\frac{\delta\Delta_{44B'}}{i_4 \cdot \sigma_{B'}} - \frac{\delta\Delta_{44B'}}{i_4 \cdot \sigma_{B'}} \right) \right\} + n_4^{-3} (n_4 - 1)(S_{26} + S_{36} - S_{45}) - (\pi_{26} + \pi_{36})/2$	(~ 32%) ~ 14%	
No. 13, 14	$\pi_{54} = \sqrt{2} n_5^{-3} \left\{ \left(\frac{\delta\Delta_{55B}}{i_5 \cdot \sigma_B} - \frac{\delta\Delta_{55B}}{i_5 \cdot \sigma_B} \right) + \left(\frac{\delta\Delta_{55B'}}{i_5 \cdot \sigma_{B'}} - \frac{\delta\Delta_{55B'}}{i_5 \cdot \sigma_{B'}} \right) \right\} + n_5^{-3} (n_5 - 1)(S_{16} + S_{36} - S_{45}) - (\pi_{16} + \pi_{36})/2$	~ 14%	T.25

Table 1 (continued)

Necessary samples (see Fig. 1)	Relationships ^b	Calculated error ^a	No. of equation
		(~ 32%)	
	$\pi_{56} = \sqrt{2}n_5^{-3} \left\{ \left(\frac{\delta\Delta_{55\bar{B}}}{i_5\sigma_{\bar{B}}} - \frac{\delta\Delta_{55B}}{i_5\sigma_B} \right) - \left(\frac{\delta\Delta_{55\bar{B}'}}{i_5\sigma_{\bar{B}'}} - \frac{\delta\Delta_{55B'}}{i_5\sigma_{B'}} \right) \right\}$ $+ n_5^{-3}(n_5 - 1)(S_{14} + S_{34} - S_{56}) - (\pi_{14} + \pi_{34})/2$	~ 14%	T.26
		(~ 32%)	
No. 15, 16	$\pi_{65} = \sqrt{2}n_6^{-3} \left\{ \left(\frac{\delta\Delta_{66\bar{B}}}{i_6\sigma_{\bar{B}}} - \frac{\delta\Delta_{66B}}{i_6\sigma_B} \right) + \left(\frac{\delta\Delta_{66\bar{B}'}}{i_6\sigma_{\bar{B}'}} - \frac{\delta\Delta_{66B'}}{i_6\sigma_{B'}} \right) \right\}$ $+ n_6^{-3}(n_6 - 1)(S_{14} + S_{24} - S_{56}) - (\pi_{14} + \pi_{24})/2$	~ 14%	T.27
		(~ 32%)	
	$\pi_{64} = \sqrt{2}n_6^{-3} \left\{ \left(\frac{\delta\Delta_{66\bar{B}}}{i_6\sigma_{\bar{B}}} - \frac{\delta\Delta_{66B}}{i_6\sigma_B} \right) - \left(\frac{\delta\Delta_{66\bar{B}'}}{i_6\sigma_{\bar{B}'}} - \frac{\delta\Delta_{66B'}}{i_6\sigma_{B'}} \right) \right\}$ $+ n_6^{-3}(n_6 - 1)(S_{15} + S_{25} - S_{46}) - (\pi_{15} + \pi_{25})/2$	~ 14%	T.28
		(~ 32%)	

^a The calculated error in the brackets corresponds to the one-fold interferometric method.

^b The directions $4'$, $4''$, $4'$, ... are shown in Fig. 1.

Finally, inserting Eq. (6) into Eq. (1) one gets two equations; the difference between them is the simplified relationship (Table 1, Eq. (T.2)) for the calculation of the constant π_{14} by the two-fold measurement method (the sign before π_{14} in (6) depends on the choice of axis sign of the crystallophysic coordinate system [5,6]). In the same way one can obtain the simplified relationships also for the constants π_{25} , π_{36} (Table 1, Eqs. (T.3), (T.4)), π_{24} , ..., π_{26} (Table 1, Eqs. (T.8)–(T.13)), as well as for the constants, for which $i > 3$, $m \leq 3$ i.e. π_{41} , π_{52} , π_{63} (Table 1, Eqs. (T.5)–(T.7)) and for the constants for which $i > 3$, $m > 3$ i.e. π_{44} , π_{55} , π_{66} (Table 1, Eqs. (T.14)–(T.16)). The latter set of constants is known to describe the rotation of the optical indicatrix under the action of shear stress.

3. Independent determination of the constants π_{42} and π_{43} , π_{51} and π_{53} or π_{61} and π_{62} can be performed by the following procedure. With the exception of the equations obtained for the direct or symmetric experimental conditions using the samples no. 2, 3, 4 (Fig. 2), it is necessary to introduce an additional equation, in which these constants would be presented. However, for the sake of simplicity we propose to make measurements on

the samples no. 5 and no. 6 (see Fig. 2 and Table 1). Considering the sample no. 5, for which the geometry of measurement is set as: $\mathbf{i} = 4'$; $\mathbf{k} = \bar{4}'$; $\mathbf{m} = 4'$ (primes indicate the directions that are turned on angle α with respect to direction $\bar{4}$ and 4) the directional cosines can be presented as

$$\begin{aligned} \alpha_{i1} &= 0, & \alpha_{k1} &= 0, & \alpha_{m1} &= 0, \\ \alpha_{i2} &= \sin \alpha, & \alpha_{k2} &= \cos \alpha, & \alpha_{m2} &= \sin \alpha, \\ \alpha_{i3} &= \cos \alpha, & \alpha_{k3} &= -\sin \alpha, & \alpha_{m3} &= \cos \alpha. \end{aligned} \quad (7)$$

Combining Eq. (7) with Eqs. (2)–(4) and Eq. (1) we obtain:

$$\begin{aligned} \pi'_{im} &= \pi_{22} \sin^4 \alpha + \pi_{33} \cos^4 \alpha \\ &+ 0.25(\pi_{23} + \pi_{32} + 2\pi_{44}) \sin^2 2\alpha \\ &+ 0.5(\pi_{24} \sin^2 \alpha + \pi_{34} \cos^2 \alpha + 2\pi_{42} \sin^2 \alpha \\ &+ 2\pi_{43} \cos^2 \alpha) \sin 2\alpha, \end{aligned} \quad (8)$$

$$\begin{aligned} S'_{km} &= 0.25(S_{22} + S_{33} - S_{44}) \sin^2 2\alpha \\ &+ S_{23}(1 - 0.5 \sin^2 2\alpha) + 0.25(S_{24} - S_{34}) \sin 4\alpha, \\ n'_i &= (n_2^{-2} \sin^2 \alpha + n_3^{-2} \cos^2 \alpha)^{-1/2}, \end{aligned}$$

from which the equation for effective piezooptic constant $\pi_{42} \sin^2 \alpha + \pi_{43} \cos^2 \alpha$ can be easily derived. In order to obtain a similar relation for the coefficients π_{42} or π_{43} with the opposite sign, the sample no. 6 should be investigated in the geometry of measurement being set as: $i = \bar{4}''$; $k = 4''$; $m = \bar{4}''$ (see Fig. 2). Then the difference between obtained relations for these symmetric measurements give again the simplified relation for calculation of effective constant $\pi_{42} \sin^2 \alpha + \pi_{43} \cos^2 \alpha$. Similar procedures must be performed for other pairs of constants, i.e. π_{51} and π_{53} or π_{61} and π_{62} . The Eqs. (T.17)–(T.22) in the Table 1 are derived for the particular case, i.e. when the angle $\alpha = 30^\circ$.

4. In order to measure the pairs of constants π_{45} and π_{46} , π_{54} and π_{56} or π_{64} and π_{65} one must prepare the sample nos. 11–16 as it is shown in the Fig. 2. As an example, here only we consider the details concerning the pair of constants π_{64} and π_{65} . In particular, the corresponding direct conditions for the experiment on sample no. 15 are: $i = 6$; $k = \bar{6}$; $m \perp B$, whereas the symmetric conditions would be then: $i = 6$; $k = \bar{6}$; $m \perp \bar{B}$. The directional cosines can be written as:

$$\begin{aligned} \alpha_{i1} &= \sqrt{2}/2, & \alpha_{k1} &= -\sqrt{2}/2, & \alpha_{m1} &= \pm 1/2, \\ \alpha_{i2} &= \sqrt{2}/2, & \alpha_{k2} &= \sqrt{2}/2, & \alpha_{m2} &= \pm 1/2, \\ \alpha_{i3} &= 0, & \alpha_{k3} &= 0, & \alpha_{m3} &= \sqrt{2}/2. \end{aligned} \quad (9)$$

Then the effective values of the piezooptic constants, elastic compliances and refractive indices can be presented as follows:

$$\begin{aligned} \pi'_{im} &= [\pi_{11} + \pi_{12} + \pi_{16} + \pi_{21} + \pi_{22} + \pi_{26} \\ &+ 2(\pi_{13} + \pi_{23} + \pi_{61} + \pi_{62} + \pi_{66}) \\ &+ 4\pi_{63} \pm \sqrt{2}(\pi_{14} + \pi_{15} + \pi_{24} + \pi_{25}) \\ &\pm 2\sqrt{2}(\pi_{64} + \pi_{65})]/8, \end{aligned} \quad (10)$$

$$\begin{aligned} S'_{km} &= [S_{11} + S_{22} - S_{66} + 2(S_{13} + S_{23} + S_{12} - S_{36}) \\ &\pm \sqrt{2}(S_{14} + S_{15} + S_{24} + S_{25} - S_{46} - S_{56})]/8, \end{aligned}$$

$$n'_i = n_6 = ((n_1^{-2} + n_2^{-2})/2)^{-1/2}.$$

Inserting (10) into (1) we obtain two required equations. By using the sample no. 16 for direct $i = \bar{6}$; $k = 6$; $m \perp B'$ and symmetric $i = \bar{6}$; $k = 6$; $m \perp \bar{B}'$ conditions of the experiment one can obtain two additional equations. Then after simple mathematical operations (subtracting in pairs) the obtained relations result to the following equations:

$$\begin{aligned} \frac{\delta\Delta_{\bar{6}\bar{6}\bar{B}}}{t_{\bar{6}} \cdot \sigma_{\bar{B}}} - \frac{\delta\Delta_{6\bar{6}B}}{t_{\bar{6}} \cdot \sigma_B} \\ = n_6^3 [\sqrt{2}(\pi_{14} + \pi_{15} + \pi_{24} + \pi_{25}) \\ + 2\sqrt{2}(\pi_{65} + \pi_{64})]/8 - [\sqrt{2}(S_{14} + S_{15} + S_{24} \\ + S_{25} - S_{46} - S_{56})](n_6 - 1)/4, \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{\delta\Delta_{\bar{6}\bar{6}B'}}{t_6 \cdot \sigma_{B'}} - \frac{\delta\Delta_{\bar{6}\bar{6}B'}}{t_6 \cdot \sigma_{B'}} \\ = n_6^3 [\sqrt{2}(\pi_{14} - \pi_{15} + \pi_{24} - \pi_{25}) \\ + 2\sqrt{2}(\pi_{65} - \pi_{64})]/8 - [\sqrt{2}(S_{14} - S_{15} + S_{24} \\ - S_{25} + S_{46} - S_{56})](n_6 - 1)/4, \end{aligned} \quad (12)$$

from which one can derive Eqs. (T.27) and (T.28) (Table 1). Thus the piezooptic constants π_{64} and π_{65} can be determined in the two-fold interferometric measurements performed on two samples. The Eqs. (T.23–T.26) concerning the piezooptic constants π_{45} and π_{46} , π_{54} and π_{56} are derived in the similar way.

Table 1 contains the set of equations that are necessary to calculate all piezooptic constants for the crystal materials of triclinic symmetry by using the data of two-fold interferometric measurements. In this case the determination of 36 components of piezooptic tensor needs to carry out 57 measurements on 16 samples. Table 2 gives additional suggestions and relations concerning the calculation procedure in crystals of all higher symmetry groups; in all cases a number of samples (or a number of required measurements) appears to be sufficiently reduced.

3. Two-fold interferometric measurements of β -BaB₂O₄ crystals

The beta barium borate crystal β -BaB₂O₄ (BBO) is known as perspective materials for nonlinear optics [7] and it is characterized by 3 m point group of symmetry [8,9]. So far we know the photoelastic properties of these crystals have never been investigated.

The measurements of piezooptic constants in BBO crystals were carried out using the experimental setup based on Mach–Zehnder interferometer [5]. In the present work, it was slightly modified in order to do simultaneous measurements of absolute piezooptic coefficients π_{im} (by two-fold interferometric method) and piezooptic constants of induced birefringence π_{km}^* (by optical-polarization method). It thus allows us to verify the validity of experimental results obtained by these independent methods; one must remember that π_{km}^* and π_{im} are related by a set of trivial equations [10]

$$\pi_{km}^* = \pi_{im} n_i^3 - \pi_{jm} n_j^3, \quad (3)$$

where n_i and n_j are the refractive indices for two orthogonal polarized light waves which propagate in \mathbf{k} direction. The pressure induced changes of optical path (interferometric method) or optical retardation (optical-polarization method) were measured by well-known half-wave stress method or by modified method of maximal intensities in which several half-wave lengths are induced. In alternative case, which is more precise,

Table 2
Supplement to Table 1: Additional relationship for the calculation of piezooptic constants for the crystals of higher symmetries

No.	Crystal system (class of symmetry)	Necessary samples	Number of measur.	No. of equation	Additional relationships and cases of their application
1	Monoclinic (all classes)	8 samples:	29	T.29	$\pi_{46} = 2\sqrt{2}n_4^{-3} \left(\frac{\delta\Delta_{44B}}{t_4\sigma_B} - \frac{\delta\Delta_{44B}}{t_4\sigma_B} \right) + n_4^{-3}(n_4 - 1)(S_{25} + S_{35} - S_{46}) - (\pi_{25} + \pi_{35})/2$ (sample no. 12 is not needed)
		no. 1; 2; 3; 4; 7; 8; 11; 15		T. 30	$\pi_{64} = 2\sqrt{2}n_6^{-3} \left(\frac{\delta\Delta_{66B}}{t_6\sigma_B} - \frac{\delta\Delta_{66B}}{t_6\sigma_B} \right) + n_6^{-3}(n_6 - 1)(S_{15} + S_{25} - S_{46}) - (\pi_{15} + \pi_{25})/2$ (sample no. 16 is not needed) π_{44}, π_{66} —according to (T.31)
2	Orthorombic (all classes)	4 samples: no. 1; 2; 3; 4	12	T.31	$\pi_{rr} = -4n_r^{-3} \frac{\delta\Delta_{rr}}{t_r\sigma_r} + n_r^{-3}(n_r - 1)(S_{ii} + S_{jj} + 2S_{ij} - S_{rr}) - (\pi_{ii} + \pi_{jj} + \pi_{ji} + \pi_{ij})/2$, where $r = 9 - i - j$ and $i \neq j = 1, 2$ or 3 (only one measurement on samples no. 2, 3 and 4 for determination of π_{44}, π_{55} and π_{66} , respectively)
3	Trigonal ($3, \bar{3}$)	5 samples: no. 1; 2; 3; 4; 11	19	T.32	$\pi_{45} = 2\sqrt{2}n_4^{-3} \left(\frac{\delta\Delta_{44B}}{t_4\sigma_B} - \frac{\delta\Delta_{44B}}{t_4\sigma_B} \right) + n_4^{-3}(n_4 - 1)(S_{25} - S_{16} - S_{45}) - (\pi_{25} - \pi_{16} + 4\pi_{52})/2$ -(sample no. 12 is not needed)
4	Trigonal ($32, 3m, \bar{3}m$)	2 samples: no. 1; 2	11		No difference
5	Tetragonal ($4, \bar{4}, 4/m$)	6 samples:	14	T.33	$\pi_{61} = \frac{4\sqrt{3}}{3} n_6^{-3} \left(\frac{\delta\Delta_{61\bar{6}6}}{t_6\sigma_6} - \frac{\delta\Delta_{61\bar{6}6}}{t_6\sigma_6} \right) - 2n_6^{-3}(n_6 - 1)S_{16} - \pi_{16}/2$ -more suitable, because $\pi_{62} = -\pi_{61}$;
		no. 1; 2; 4; 9; 10; 11		π_{45} —according to (T.32), but if takes into account that: $\pi_{25} = \pi_{52} = 0$ and $S_{25} = S_{45} = 0$; π_{44} —according to (T.31) π_{44}, π_{66} —according to T.31	
6	Tetragonal ($422, 4mm, 4/mmm$)	3 samples: no. 1; 2; 4	7		π_{44}, π_{66} —according to T.31
7	Hexagonal ($6, \bar{6}, 6/m$)	4 samples: no. 1; 2; 4; 11	10		π_{45}, π_{44} -analogously to sample no. 5 of tetragonal system
8	Hexagonal ($622, 6/mmm, 6mm, \bar{6}m2$)	2 samples: no. 1; 2	6		π_{44} —according to (T.31)
9	Cubic ($23, m\bar{3}$)	2 samples: no. 1; 2	4		π_{44} —according to (T.31)
10	Cubic ($432, 43m, m\bar{3}m$)	2 samples: no. 1; 2	3		π_{44} —according to (T.31)

the changes of the optical path were measured by method of interferometric fringe shift registration whereas the optical retardation has been determined by well-known Senarmont method.

The measurements were held at room temperature using a He–Ne laser ($\lambda = 632.8$ nm). The magnitudes of the piezooptic constants π_{im} are given in Tables 3 and 4 together with the values of their errors. The latter were calculated according to [11], i.e. taking into account meansquare error of measured mechanical stress (relative error of this measurement was estimated as 1%) and the interference fringes shift error (registration accuracy—0.02 of the bandwidth). The absolute error of the Senarmont method was about 0.1° . The results of measurements on X/45°-cut samples for β -BBO crystals are given in Table 3. Regarding the data obtained by the one-fold and two-fold interferometric methods it is obvious that the latter one is more accurate in determination of π_{14} , π_{41} , π_{44} constants. This conclusion

one obtains comparing the errors in both cases which have been estimated theoretically.

For the calculation of photoelastic constants one can use the known relation

$$p_{in}^E = \pi_{im}^E C_{mn} = \pi_{im}^E S_{mn}^{-1}, \tag{13}$$

where C_{mn} are the elastic constants and π_{im}^E are the piezooptic constants at the constant electric field E . The piezoelectric addition to elastic compliances of BBO crystals is less than 1%, therefore it can be neglected and one may consider that $S_{mn}^D = S_{mn}^E$.

The values of π_{im}^E of BBO crystals are presented in Table 4. They were calculated using the known equation

$$\pi_{im}^E \equiv \pi_{\lambda\mu\eta\nu}^E = \pi_{\lambda\mu\eta\nu}^D - \frac{r_{\lambda\mu\tau} d_{\tau\eta\nu}}{\epsilon_0(\epsilon_{\tau\tau} - 1)}, \tag{14}$$

where the second term represents a secondary electro-optic addition, $r_{\lambda\mu\tau}$ and $d_{\tau\eta\nu}$ are the linear electrooptic constants and piezoelectric modules, respectively, $\epsilon_0 =$

Table 3
The data of interferometric measurements performed on samples of X/45°-cut of β -BaB₂O₄ crystals

No*	Sample geometry (in direct and symmetric condition ^a)			Effective values according to Eqs. (2) and (3)	Measured values of π_{im} , Br = 10^{-12} m ² /N	
	<i>k</i>	<i>m</i>	<i>I</i>		One-fold method	Two-fold method (Table 1)
1	2			3	4	5
1	4	1	$\bar{4}$	$\pi'_{41} = (\pi_{21} + \pi_{31} - 2\pi_{41})/2$ $S'_{41} = (S_{12} + S_{13} + S_{14})/2$	$\pi_{41} = -2.0 \pm 0.10$	$\pi_{41} = -2.03 \pm 0.07$
1a	$\bar{4}$	1	4	$\pi'_{41} = (\pi_{21} + \pi_{31} + 2\pi_{41})/2$ $S'_{41} = (S_{12} + S_{13} - S_{14})/2$	$\pi_{41} = -2.0 \pm 0.17$	
2	4	$\bar{4}$	1	$\pi'_{14} = (\pi_{12} + \pi_{13} - \pi_{14})/2$ $S'_{44} = (S_{22} + S_{33} + 2S_{23} - S_{44})/2$	$\pi_{14} = -2.0 \pm 1.3$	$\pi_{14} = -2.0 \pm 0.8$
2a	$\bar{4}$	4	1	$\pi'_{14} = (\pi_{12} + \pi_{13} + \pi_{14})/2$ $S'_{44} = S'_{44}$	$\pi_{14} = -2.0 \pm 1.2$	
3	4	$\bar{4}$	$\bar{4}$	$\pi'_{44} = (\pi_{22} + \pi_{23} + \pi_{32} + \pi_{33} - \pi_{24} - 2\pi_{42} + 2\pi_{44})/4$ S'_{44}	$\pi_{44} = -26 \pm 1.4$	$\pi_{44} = -26.3 \pm 0.9$
3a	$\bar{4}$	4	4	$\pi'_{44} = (\pi_{22} + \pi_{23} + \pi_{32} + \pi_{33} + \pi_{24} + 2\pi_{42} + 2\pi_{44})/4$ S'_{44}	$\pi_{44} = -27 \pm 1.5$	

^a 1, 2, 3—direct conditions; 1a, 2a, 3a—symmetric conditions.

Table 4
The average values of piezooptical coefficients at a constant displacement $D(\pi_{im}^D)$, at a constant electric field $E(\pi_{im}^E)$ and the calculated values of photoelastic constants p_{in}^E of the β -BaB₂O₄ crystals

Constant, contribution	Indices <i>im</i> or <i>in</i>							
	11	12	13	31	33	14	41	44
π_{im}^D , Br	-1.7 ± 0.15	-1.35 ± 0.08	1.75 ± 0.23	-1.6 ± 0.15	3.7 ± 0.4	-2.0 ± 0.8	-2.03 ± 0.07	-26.3 ± 0.9
Electrooptic contribution, Br	-0.12	0.11	0.02	-0.01	0.02	-0.46	-0.01	-0.05
π_{im}^E , Br	-1.6	-1.46	1.73	-1.6	3.7	-1.54	-2.02	-26.3
p_{in}^E	-0.195	-0.197	-0.059	-0.112	0.039	-0.005	-0.007	-0.078

$8.85 \cdot 10^{-12}$ F/m and $\varepsilon_{\tau\tau}$ is the dielectric constant of the crystal. The following values have been used in our calculations: $r_{113} = 0,27$; $r_{222} = -2,41$; $r_{333} = 0,29$; $r_{131} = 1,7$ and $d_{311} = -1,17$; $d_{222} = 2,30$; $d_{333} = 3,4$; $d_{113} = -9,6$ (all values are in 10^{-12} m/V) [12], and $\varepsilon_{11} = \varepsilon_{22} = 6,7$, $\varepsilon_{33} = 8,1$ [8]. In most cases the secondary electrooptic addition, which is less than 1%, can be neglected (see Table 4). At the same time a considerable electrooptic contribution is revealed for the piezooptic constants π_{11} , π_{22} (about 10%) and π_{14} (about 23%) which indeed is a sequence of a large magnitude of r_{222} and d_{113} constants. It is quite obvious that in this case the electrooptic correction cannot be neglected since it may cause the significant errors in calculation of the photoelastic constants p_{in}^E . The magnitudes of photoelastic constants p_{in}^E and piezoelectric constants π_{in}^E of BBO crystals are presented in Table 4. For these calculations the data concerning the elastic compliances S_{mm} were taken from [8], except the constant $S_{14} = 22.6$ Br, which was estimated in our recent measurements. As one can see from Table 4, BBO crystals have relatively large values of photoelastic constants, especially for p_{11} and p_{12} . Accordingly, these crystals have an advantage comparing to such well-known acoustooptic materials as LiNbO_3 or TeO_2 ; their photoelastic (piezooptic) constants are comparable to the corresponding constants of fused quartz.

4. Conclusions

In conclusion we present here the interferometric technique which allows to measure piezooptical and photoelastic characteristics of crystal materials of any symmetry. The offered two-fold interferometric method enables to determine all independent non-zero piezooptic and photoelastic constants by measuring piezo-induced changes of optical path. As advantage to known acoustooptical techniques this method allows to measure both the absolute magnitude and sign of photoelastic constants. In general case the determination of 36 components of piezooptic tensor needs to carry out 57 measurements on 16 samples. The corresponding relationships are derived.

As an example we apply here the interferometric technique to measure the piezooptic and photoelastic

constants in trigonal β - BaB_2O_4 crystals. Taking into account the secondary electrooptical contributions to piezooptical constants at constant electrical displacement D , the sign and the magnitude of all photoelastic constants are determined. We have revealed relatively large values of photoelastic constants in BBO crystals, especially for p_{11} and p_{12} components. Accordingly, BBO crystals can be considered as perspective materials for applications in piezooptical sensors.

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