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Spatial anisotropy of linear electro-optic effect in crystal materials: II. Indicative surfaces as efficient tool for electro-optic coupling optimization in LiNbO₃

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ABSTRACT

The electro-optic (EO) properties of pure and MgO-doped LiNbO₃ crystals are characterized by indicative surfaces (IS) describing the spatial anisotropy of the longitudinal and transverse linear EO effects. By analyzing the IS and their stereographic projections, we have determined the geometries with maximum EO coupling. The maximum magnitudes of the linear EO effect are revealed in the longitudinal and transverse geometries along the directions that substantially deviate from the principal crystallographic axes of both crystals. Spatial analysis of the IS thus yields a set of the optimized sample geometries recommended for designers who develop EO modulators or deflectors. Just by switching from the standard cell geometry to the optimized cell geometry, as determined in the present work, one may improve almost three times the modulation efficiency of EO devices based on lithium niobate crystals. This results in a corresponding reduction of their driving voltages being evidently of great practical importance for many applications.

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1. Introduction

Along with the development of optoelectronics, much efforts have been expended for the past several decades to create highefficiency electro-optic (EO) cells that can be operated by low driving voltages. A required modulation efficiency of such devices is usually tried to be achieved through a search for highperformance EO crystals. At the same time, their modulation efficiency in many cases may be considerably improved just by choosing an appropriate geometry of the EO coupling.

In our previous paper [1], we have derived the equations suitable for the determination of a complete set of linear EO tensor components in crystal materials of any symmetry. The method is based on the interferometry technique and has been tested on lithium niobate crystals as well as applied to Mg-doped LiNbO₃ crystals to study their EO properties. However, having even a complete set of linear EO coefficients, it is not so trivial to find in each particular case an *optimized geometry* of EO cell. Under the cell geometry (or geometry of the EO coupling), we imply the

* Corresponding author. E-mail address: andriy.kityk@univie.ac.at (A.V. Kityk). crystallographic orientation of the cell being usually cut from the EO crystal in a shape of a rectangular prism with the faces that are oriented according to certain crystallographic directions, not necessary to the principal ones. These crystallographic directions are bonded to an applied electric field, direction of light propagation and its polarization and, therefore, the cell geometry defines indeed the geometry of the EO coupling. The optimized cell geometry, which corresponds to the maximum EO coupling, allows one to achieve the best modulation efficiency of the EO cells fabricated from the same crystal materials. Up to now, the geometry optimization regarding the EO cells has been ignored by many authors. Even for the uniaxial crystals, such as classic LiNbO₃, this problem has been never considered. In standard applications, the EO cells up to now are usually made of their socalled direct cuts being fabricated in a shape of rectangular prisms with the faces that are perpendicular to the principal crystallographic axes of the LiNbO₃ crystal. According to the analysis given in this work, the cell geometry chosen in such a way is not optimal. By using, e.g., non-direct cuts of a certain crystallographic orientation, as will be determined in the present work, one may achieve the modulation efficiency that is several times higher compared to the one obtained for the cells fabricated from the direct cuts of LiNbO₃ crystals. The spatial anisotropy analysis of the photoelastic effect has been considered in several of our

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recent papers [2,3]. In particular, by means of computer simulations based on the optimization procedure, the cell geometries with the best acoustooptic efficiency (figure of merit) were determined for the acoustooptic cells made of β -BaB₂O₄ [2] and Cs₂HgCl₄ [3] crystals.

The present paper deals with the spatial anisotropy analysis regarding the linear EO effect in crystal materials. The main purpose of such a study is to determine the geometry of the most efficient EO coupling. Similar to Refs. [2,3], the analysis is based on the indicative surfaces (IS) and is applied to pure and MgO-doped LiNbO₃ single crystals. However, using the metho-dology developed in this article, the optimized geometry can be in principle found for any other crystal materials, including the low symmetry ones. This explains our interest in this problem, being important for optical engineering, which deals with the designing of EO devices.

2. IS of the linear EO effect in LiNbO₃ and LiNbO₃:MgO crystals

Three-dimensional analysis of spatial anisotropy of the linear EO effect is based on the IS and their stereographic projections. The linear EO effect is described by the third-rank tensor. In arbitrary Cartesian coordinate system, the linear EO tensors r'_{unh} (where u, n, h = 1, 2 or 3 are the indices each corresponding to one of the orthogonal axes X'_{1} , X'_{2} or X'_{3}) are related to the tensor components r_{pcq} in the principal crystallophysical system (where p, c, q = 1, 2 or 3 are the indices each corresponding to one of the principal axes X_{1}, X_{2} or X_{3}) by the relation [4]

$$r'_{unh} = \alpha_{up} \alpha_{nc} \alpha_{hq} r_{pcq},\tag{1}$$

where α_{up} , α_{nc} , α_{hq} are the directional cosines of the rotated coordinate system X'_{1} , X'_{2} , X'_{3} with respect to the principal coordinate system X_1 , X_2 , X_3 (see Fig. 1). In order to obtain the IS, one must calculate the tensor components r'_{unh} as the function of the directional cosines α_{up} , α_{nc} , α_{hq} . For the linear EO effects, the axes of the arbitrary rotated coordinate system are bonded to the direction of light propagation and its polarizations, which indeed decreases the number of IS [2,5]. We denote **k** as the direction of light propagation; **r** the radius-vector of IS, which can be parallel to one of the permitted light polarizations i; direction j coincides with the alternative light polarization being orthogonal to the vectors **k** and **i**. The vectors **i**, **j** or **k** are chosen to be parallel to the axes of the rotated coordinate system X'_1 , X'_2 or X'_3 , respectively, as shown in Fig. 1. The directional cosines yield thereby a matrix of orthogonal transformation [4] from the principal crystallophysical coordinate system to the arbitrary rotated one. In spherical coordinates *r*, θ and ϕ the directional cosines of the radius-vector **r** take the forms (see Fig. 1):

$$\alpha_{r1} = \sin \theta \cos \varphi, \quad \alpha_{r2} = \sin \theta \sin \varphi, \quad \alpha_{r3} = \cos \theta.$$
 (2)

thus for the uniaxial crystals, the directional cosines for the light polarization vector of the extraordinary wave **j** and the vector



Fig. 1. Spatial position of the rotating coordinate system X'_1, X'_2, X'_3 with respect to the crystallophysical coordinate system X_1, X_2, X_3 for uniaxial crystals (**r** is the radius-vector of IS; **k**, **i** and **j** are the directions of light propagation and its two possible polarizations, respectively).

of its propagation **k** may be written as follows:

$$\alpha_{j1} = -\sin \varphi, \quad \alpha_{j2} = \cos \varphi, \quad \alpha_{j3} = 0; \tag{3}$$

$$\alpha_{k1} = \cos \theta \cos \varphi, \quad \alpha_{k2} = \cos \theta \sin \varphi, \quad \alpha_{k3} = -\sin \theta.$$
 (4)

Then the IS for principal components $r'_{i\ell}(i,\ell = 1, 2, 3)$ of the linear EO effect in most general case of the lowest triclinic symmetry is described by the equation

$$\begin{aligned} r'_{i\ell} &= \alpha_{i1}^2 \alpha_{\ell 1} r_{11} + \alpha_{i1}^2 \alpha_{\ell 2} r_{12} + \alpha_{i1}^2 \alpha_{\ell 3} r_{13} + \alpha_{i2}^2 \alpha_{\ell 1} r_{21} \\ &+ \alpha_{i2}^2 \alpha_{\ell 2} r_{22} + \alpha_{i2}^2 \alpha_{\ell 3} r_{23} + \alpha_{i3}^2 \alpha_{\ell 1} r_{31} \\ &+ \alpha_{i3}^2 \alpha_{\ell 2} r_{32} + \alpha_{i3}^2 \alpha_{\ell 3} r_{33} + 2 \alpha_{i2} \alpha_{i3} \alpha_{\ell 1} r_{41} + 2 \alpha_{i2} \alpha_{i3} \alpha_{\ell 2} r_{42} \\ &+ 2 \alpha_{i2} \alpha_{i3} \alpha_{\ell 3} r_{43} + 2 \alpha_{i1} \alpha_{i3} \alpha_{\ell 1} r_{51} \\ &+ 2 \alpha_{i1} \alpha_{i3} \alpha_{\ell 2} r_{52} + 2 \alpha_{i1} \alpha_{i3} \alpha_{\ell 3} r_{53} + 2 \alpha_{i1} \alpha_{i2} \alpha_{\ell 1} r_{61} \\ &+ 2 \alpha_{i1} \alpha_{i2} \alpha_{\ell 2} r_{62} + 2 \alpha_{i1} \alpha_{i2} \alpha_{\ell 3} r_{63}, \end{aligned}$$
(5)

where taking into account the permutation symmetry of the linear EO tensor $r_{i\ell}$ (ℓ is the direction of applied electric field **E**) with respect to the two first indices, we have replaced them by the one index using the Voight notation (i.e. 1=11, 2=22, 3=33, 4=23, 5=13 and 6=12); thus, $r_{i\ell} = r_{pc\ell}$ for $pc \leftrightarrow i = 1, ..., 6$ [4,6]. In the case of LiNbO₃ crystals (point group symmetry 3m), Eq. (5) can then be reduced to the form

$$\begin{aligned} \mathbf{r'}_{i\ell} &= ((\alpha_{i2}^2 - \alpha_{i1}^2)\alpha_{\ell 2} - 2\alpha_{i1}\alpha_{i2}\alpha_{\ell 1})\mathbf{r}_{22} + (\alpha_{i1}^2 + \alpha_{i2}^2)\alpha_{\ell 3}\mathbf{r}_{13} \\ &+ \alpha_{i3}^2\alpha_{\ell 3}\mathbf{r}_{33} + 2\alpha_{i3}(\alpha_{i1}\alpha_{\ell 1} + \alpha_{i2}\alpha_{\ell 2})\mathbf{r}_{51}. \end{aligned}$$

Thinking of the photoelastic effect, it is usually characterized by the three IS that represent the geometries frequently used in most practical applications [2,3,5,7]. For the linear EO effect, in addition to these three IS, one can construct also the IS corresponding to the case when $\mathbf{k} \parallel \boldsymbol{\ell}$ (see Eq. (10)). Therefore, by inserting Eqs. (2)–(4) into (6) one obtains the following four equations for the IS of linear EO effect in the spherical coordinate system:

$$r'_{ii}(\theta, \varphi) = -r_{22}\sin^3\theta\sin 3\varphi + (r_{13} + 2r_{51})\sin^2\theta\cos\theta + r_{33}\cos^3\theta,$$
(7)

$$r_{i\ell}^{\prime(i)}(\theta,\varphi) = -r_{22}\sin^2\theta\cos3\varphi,\tag{8}$$

$$r_{i\ell}^{\prime(\ell)}(\theta,\varphi) = r_{22}\sin\theta\sin^2\theta + r_{13}\cos\theta, \qquad (9)$$

 $r_{i\ell}^{(k)}(\theta,\varphi) = -r_{22} \sin \theta \cos^2 \theta \sin 3\varphi + r_{13} \cos^3 \theta + (r_{33} - 2r_{51}) \sin^2 \theta \cos \theta.$ (10)

Here the IS $r'_{ii}(\theta, \varphi)$ corresponds to the geometry: $\mathbf{r} \| \mathbf{i} \| \boldsymbol{\ell} \| X'_3, \mathbf{k} \| X'_1$ (see Fig. 1.), i.e. it represents the longitudinal ($\mathbf{i} \| \boldsymbol{\ell}$) EO effect. The IS $r'_{i\ell}^{(i)}(\theta, \varphi)$ ($\mathbf{r} \| \mathbf{i} \| X'_3, \boldsymbol{\ell} \| X'_2 \mathbf{k} \| X'_1$) or $r'_{j\ell}^{(i)}(\theta, \varphi)$ ($\mathbf{r} \| \boldsymbol{\ell} \| X'_3, \mathbf{j} \| X'_2, \mathbf{k} \| X'_1$) and $r'_{i\ell}^{(k)}(\theta, \varphi)$ ($\mathbf{r} \| \mathbf{k} \| \boldsymbol{\ell} \| X'_3, \mathbf{i} \| X'_1$) describe the IS of the transverse (\mathbf{i} or $\mathbf{j} \perp \boldsymbol{\ell}$) EO effect for the two possible orthogonal light polarizations \mathbf{i} or \mathbf{j} , respectively. The case $\mathbf{k} \| \boldsymbol{\ell}$ for the polarization \mathbf{j} is fully described by IS $r'_{j\ell}^{(\ell)}(\theta, \varphi)$ converting θ to θ +90°. The superscript index in the brackets indicates the direction being parallel to the radius-vector of the IS describing the transverse EO effect.

Fig. 2 presents the IS and their stereographic projections for the longitudinal $r'_{i\ell}(\theta,\varphi)$ [(a)] and transverse $r'_{i\ell}^{(\ell)}(\theta,\varphi)$ [(b)], $r'_{i\ell}^{(\ell)}(\theta,\varphi)$ [(c)] and $r'_{i\ell}^{(k)}(\theta,\varphi)$ [(d)] linear EO effect of LiNbO₃ crystals as calculated by means of Eqs. (7)–(10). The magnitudes of the "unclamped" EO tensor components used in this evaluation have been determined in our previous work [1], and are given in Ref. [8]. More details concerning the technique used in a drawing of the IS are given in Refs. [2,5]. The extreme values for each IS of LiNbO₃ are given in Table 1. It should be emphasized that the minima with zero magnitudes of the linear EO effect occur at arbitrary angles φ , and at certain angles θ , whereas Tables 1 and 2 contain only the angular data regarding the principal crystallographic plain, i.e. for angles $\varphi = 30+n \cdot 60^\circ$. The IS has no

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Fig. 2. The IS of the linear EO effect and their stereographic projections in LiNbO₃ crystals: (a) IS $r'_{it}(\theta, \varphi)$ of the longitudinal EO effect [**r**||**i**||*ℓ*||*X*'₃, **k**||*X*'₁]; (b) IS $r'_{it}^{(0)}(\theta, \varphi)$ of the transverse EO effect [**r**||**i**||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*K*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)}(\theta, \varphi)$ of the transverse EO effect [**r**||**k**||*ℓ*||*X*'₃, **k**||*X*'₁]; (c) IS $r'_{it}^{(\ell)$

Table 1

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Extreme values for the IS of linear EO effect in crystals of LiNbO₃ and LiNbO₃:MgO (T = 300 K; $\lambda = 632.8$ nm)

IS	Minimal magnitudes for the IS and its direction			Maximum magnitudes for the IS and its direction		
	Magnitude (10 ⁻¹² m/V)	θ (deg)	ϕ (deg)	Magnitude (10 ⁻¹² m/V)	θ (deg)	φ (deg)
$r'_{ii}(\theta, \varphi)$, Fig. 2(a), (Fig. 3(a))	0	85	30, 150, 270	39.7 [43.3]	43 [45]	90, 210, 330
	0	95	90, 210, 330	–39.7 [–43.3]	137 [135]	30, 150, 270
$r'_{il}^{(i)}(\theta, \varphi)$, Fig. 2(b), (Fig. 3(b))	0	Arbitrary	30, 90, 150	6.79 [7.47]	90	60, 180, 300
	0	Arbitrary	210, 270, 330	-6.79 [-7.47]	90	0, 120, 240
$r'_{j\ell}^{(\ell)}(\theta, \varphi)$, Fig. 2(c), (Fig. 3(c))	0	56	90, 210, 330	12.2 [13.2]	34	30, 150, 270
	0	124	30, 150, 270	-12.2 [-13.2]	146	90, 210, 330
$r'_{i\ell}^{(k)}(\theta, \varphi)$, Fig. 2(d), (Fig. 3(d))	0 0 0	26 [25] 36 [34] 90	30, 150, 270 90, 210, 330 Arbitrary	11.2 [13.7] –11.2 [–13.7]	123 57	90, 210, 330 30, 150, 270

The values in the brackets are given for LiNbO₃:MgO, only if they differ from the corresponding magnitudes of pure LiNbO₃.

rotation symmetry, which is evident from Fig. 2 and agrees with German's theorem [9]. Moreover, these surfaces are characterized by the point group symmetry 3m in accordance with Neumann's principle [10]. The existence of three-fold axis is also evident from Eqs. (7)–(10), according to the angular factors $\cos 3\varphi$ or $\sin 3\varphi$. The maximum magnitudes of the EO effect do not appear along the principal crystallographic axes, but in the directions that are turned out from the optical axis (X_3) on the angle $\theta_m = 43^\circ$ (for $\varphi_m = 90^\circ$, 210° or 330°) or $\theta_m = 137^\circ$ (for $\varphi_m = 30^\circ$, 150° or 270°). The effective magnitude of the EO constant at these angles $[|r'_{ii}(\theta_m, \varphi_m)| = 39.7 \times 10^{-12} \text{ m/V}]$ exceeds about 20% the magnitude of the EO coefficient $r_{33} = 33.2 \times 10^{-12} \text{ m/V}$, the largest one

being in the set of EO tensor components for LiNbO₃ crystals [8]. For comparison, the transverse EO effect reveals much weaker extreme values on its IS $[|r_{j\ell}^{(\ell)}(\theta_m, \varphi_m)| = 12.2 \times 10^{-12} \text{ m/V}]$, i.e. it is more than three times smaller compared with the longitudinal EO effect and appear at the angles $\theta_m = 34^\circ$ ($\varphi_m = 30^\circ$, 150° or 270°) or $\theta_m = 146^\circ$ ($\varphi_m = 90^\circ$, 210° or 330°).

In a similar way, we have obtained the IS of MgO-doped LiNbO₃ crystals. Fig. 3 presents only the stereographic projections of IS for the longitudinal $r'_{ii}(\theta,\varphi)$ [(a)] and transverse $r'_{i\ell}^{(i)}(\theta,\varphi)$ [(b)], $r'_{j\ell}^{(\ell)}(\theta,\varphi)$ [(c)] and $r'_{i\ell}^{(k)}(\theta,\varphi)$ [(d)] linear EO effect for MgO-doped crystals. Since the magnitudes of the linear EO coefficients of LiNbO₃:MgO crystals do not differ considerably from the

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Table 2

The extreme values for the IS of EIPR in crystals of LiNbO₃ and LiNbO₃:MgO (T = 300 K; $\lambda = 632.8$ nm)

IS	Minimal magnitudes for the IS and its direction			Maximum magnitudes for the IS and its direction		
	Magnitude (10 ⁻¹² m/V)	θ (deg)	ϕ (deg)	Magnitude (10 ⁻¹² m/V)	θ (deg)	φ (deg)
$r'_{k\ell}^{o(\ell)}(\theta, \phi)$, Fig. 4(a), (Fig. 3(e))	0	78	30, 150, 270	432 [475]	54	90, 210, 330
	0	102	90, 210, 330	-432 [-475]	126	30, 150, 270
$r'_{k\ell}^{o(i)}(\theta, \phi)$, Fig. 4(b), (Fig. 3(f))	0	Arbitrary	30, 90, 150	162 [178]	90	60, 180, 300
	0	Arbitrary	210, 270, 330	-162 [-178]	90	0, 120, 240
$r'_{k\ell}^{o(k)}(\theta, \phi)$, Fig. 4(c), (Fig. 3(g))	0	6, 17, 79 [7, 14, 80]	90, 210, 330	263 [303]	124	90, 210, 330
	0	101, 163, 174 [100, 166, 173]	30, 150, 270	–263 [–303]	56	30, 150, 270

The values in the brackets are given for the LiNbO₃:MgO, only if they differ from the corresponding magnitudes of pure LiNbO₃.



Fig. 3. The stereographic projections for the IS of the linear EO effect in MgO-doped LiNbO₃ crystals: (a) for IS $r'_{il}(\theta, \varphi)$ [**r**||**i**|| ℓ | X'_3 , **k**|| X'_1]; (b) for IS $r'_{il}^{(l)}(\theta, \varphi)$ [**r**||**i**|| ℓ | X'_3 , **k**|| X'_1]; (c) for IS $r'_{il}^{(l)}(\theta, \varphi)$ [**r**||**i**|| ℓ | X'_3 , **j**|| X'_2 , **k**|| X'_1]; (d) for IS $r'_{il}^{(l)}(\theta, \varphi)$ [**r**||**i**|| ℓ || X'_3 , **i**|| X'_1]; (e) for IS of EIPR $r'_{kc}^{(c')}(\theta, \varphi)$ [**r**||**i**|| ℓ || X'_3 , **j**|| X'_2 , **k**|| X'_1]; (f) for IS of EIPR $r'_{kc}^{(c')}(\theta, \varphi)$ [**r**||**i**|| ℓ || X'_3 , **j**|| X'_2 , **k**|| X'_1]; (f) for IS of EIPR $r'_{kc}^{(c')}(\theta, \varphi)$ [**r**||**i**|| ℓ || X'_3 , **j**|| X'_2 , **k**|| X'_1]; (f) for IS of EIPR $r'_{kc}^{(c')}(\theta, \varphi)$ [**r**||**i**|| ℓ || X'_3 , **j**|| X'_2 , **k**|| X'_1]; (f) for IS of EIPR $r'_{kc}^{(c')}(\theta, \varphi)$ [**r**||**i**|| ℓ || X'_3 , **j**|| X'_2 , **k**|| X'_1]; (f) for IS of EIPR $r'_{kc}^{(c')}(\theta, \varphi)$ [**r**||**i**|| ℓ || X'_3 , **j**|| X'_2 , **k**|| X'_1]; (f) for IS of EIPR $r'_{kc}^{(c')}(\theta, \varphi)$ [**r**||**i**|| ℓ || X'_3 , **j**|| X'_2 , **k**|| X'_1]; (f) for IS of EIPR $r'_{kc}^{(c')}(\theta, \varphi)$ [**r**||**i**|| ℓ || X'_3 , **j**|| X'_2 , **k**|| X'_1]; (f) for IS of EIPR $r'_{kc}^{(c')}(\theta, \varphi)$ [**r**||**i**|| ℓ || X'_3 , **j**|| X'_2 , **k**|| X'_1]; (f) for IS of EIPR $r'_{kc}^{(c')}(\theta, \varphi)$ [**r**||**i**|| ℓ || X'_3 , **j**|| X'_2 , **k**|| X'_1]; (f) for IS of EIPR $r'_{kc}^{(c')}(\theta, \varphi)$ [**r**||**i**|| ℓ || X'_3 , **j**|| X'_2 , **k**|| X'_1]; (f) for IS of EIPR $r'_{kc}^{(c')}(\theta, \varphi)$ [**r**||**i**|| ℓ || X'_3 , **j**|| X'_2 , **k**|| X'_1].

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ones of pure LiNbO₃ [8], their IS appear to be very similar and, therefore, they are not shown in Fig. 3. Accordingly, the angles at which the extreme values of longitudinal or transverse linear EO effects occur on their IS are close to each other for the pure and MgO-doped crystals (see Table 1). In particular, the maximum magnitudes of the longitudinal EO effect $[|r'_{ii}(\theta_m, \varphi_m)| = 43.3 \times 10^{-12} \text{ m/V}]$ appears along the directions that are turned out from the optical axis on the angles $\theta_m = 45^{\circ}$ ($\varphi_m = 90^{\circ}$, 210° or 330°) or $\theta_m = 135^{\circ}$ ($\varphi_m = 30^{\circ}$, 150° or 270°). For comparison, the maximum value of the transverse EO effect is again much weaker $[|r'_{il}(k)(\theta_m, \varphi_m)| = 13.7 \times 10^{-12} \text{ m/V}]$ and corresponds to the directions defined by the angles $\theta_m = 57^{\circ}$ ($\varphi_m = 30^{\circ}$, 150° or 270°) or $\theta_m = 123^{\circ}$ ($\varphi_m = 90^{\circ}$, 210° or 330°). Thus the doping of LiNbO₃ crystals by MgO does not lead to a

substantial improvement of their EO properties. However, it rises by about four times the resistance of these crystal materials [11] with respect to a powerful laser radiation, which may be important for many applications.

The extreme values on the IS may be found by applying the conditions $\partial r/\partial \theta = 0$ and $\partial r/\partial \varphi = 0$. The corresponding procedure is similar to what has been applied for the spatial analysis of the piezo-optical properties in lithium niobate [5], β -BaB₂O₄ [12] or Cs₂HgCl₄ [3] crystals. By differentiating Eq. (7) with respect to θ , one obtains for the longitudinal effect:

$$\frac{\partial}{\partial \theta} r'_{ii}(\theta, \varphi) = -\sin^2 \theta(a(\varphi)\cos\theta + b\sin\theta) - c\cos^2 \theta\sin\theta, \quad (11)$$

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where $a(\varphi) = 3r_{22}\sin 3\varphi$, $b = r_{13}+2r_{51}$ and $c = 3r_{33}-2r_{13}-4r_{51}$. Therefore, the extreme values can be found by solving the equations $\sin \theta(a(\varphi)\cos \theta+b\sin \theta)+c\cos^2\theta=0$ and $\sin \theta=0$ and occur at $\theta_{\text{extr}} = \pm n\pi$ (n = 0, 1, 2, ...) and

$$\tilde{\theta}_{\text{extr}} = \arctan\left(-\frac{a(\varphi)}{2b} \pm \frac{\sqrt{a^2(\varphi) - 4cb}}{2b}\right).$$
(12)

On the other hand, from the condition $\frac{\partial}{\partial \varphi} r'_{ii}(\theta, \varphi) = -3r_{22} \sin^3 \theta \cos 3\varphi = 0$ we obtain $\varphi = \pm (\pi/6) + (2\pi/3)n$ (n = 0, 1, 2, ...). By means of Eq. (12), one can calculate $\tilde{\theta}_{\text{extr}}$ for different values of φ , in particular, for $\varphi = \pi/6$ it takes the form

$$\tilde{\theta}_{\text{extr}} = \arctan\left(-\frac{3r_{22}}{2(r_{13}+2r_{51})} \pm \frac{\sqrt{9r_{22}^2 - 4(3r_{33}-2r_{13}-4r_{51})(r_{13}+2r_{51})}}{2(r_{13}+2r_{51})}\right),\tag{13}$$

which yields $\tilde{\theta}_{extr} = 43^{\circ}$ and 45° for the pure and MgO-doped LiNbO₃ crystals, respectively. These angular magnitudes appear in good agreement with corresponding values as obtained by means of computer calculations (see Table 1). In a similar way, one obtains from Eq. (8)

$$\frac{\partial}{\partial \theta} r_{i\ell}^{\prime(i)}(\theta, \varphi) = -r_{22} \cos 3\varphi \sin 2\theta = 0 \Rightarrow \theta = n\pi/2$$

and $\varphi = \pm \pi/6 + 2\pi n/3$,
$$\frac{\partial}{\partial \theta} r_{i\ell}^{\prime(i)}(\theta, \varphi) = 3r_{22} \sin 3\varphi \sin^2 \theta = 0 \Rightarrow \theta = \pm \pi n$$

and $\varphi = \pi n/3$, (14)

and from Eq. (9)

$$\frac{\partial}{\partial \theta} r_{j\ell}^{\prime(\ell)}(\theta, \varphi) = r_{22} \sin 3\varphi \cos \theta - r_{13} \sin \theta = 0$$

$$\Rightarrow \theta = \arctan\left(\frac{r_{22}}{r_{13}} \sin 3\varphi\right) \pm \pi n,$$

$$\frac{\partial}{\partial \varphi} r_{i\ell}^{\prime(\ell)}(\theta, \varphi) = 3r_{22} \sin \theta \cos 3\varphi = 0$$

$$\Rightarrow \varphi = \pm \pi/6 + 2\pi n/3,$$
(15)

where n = 0, 1, 2, ... In the last case, if $\varphi = \pi/6$ the angle $\theta_{\text{extr}} = 34^{\circ}$. The angular magnitudes obtained by means of Eqs. (14) and (15) are again in good agreement with the corresponding extreme values obtained within the numerical optimization (see Table 1).

3. IS of electro-induced phase retardation

For designing EO cells, it is important to have the IS of the electro-induced phase retardation (EIPR) rather than the IS of the EO effect itself, which have been considered in the previous section. The IS of EIPR corresponds to a change of the phase retardation between the orthogonally polarized waves travelling in the same direction, which is induced by an applied electric field. It is usually characterized by the effective EO coefficients [13,14]:

$$r_{k\ell}^{0} = r_{i\ell} n_{i}^{3} - r_{i\ell} n_{i}^{3} - 2(n_{i} - n_{j}) d_{\ell k},$$
(16)

where $d_{\ell k}$ is the tensor of the inverse piezoelectrical effect given here in a matrix representation. In order to get the equations for the IS of EIPR, the magnitudes $r_{i\ell}$, $r_{j\ell}$, n_i and $d_{\ell k}$ should be replaced by their angular dependencies $r'_{i\ell}(\theta,\varphi)$ (i.e. the IS of linear EO effect), $n'_i(\theta,\varphi)$ (i.e. the optical indicatrix) and $d'_{\ell k}(\theta,\varphi)$ (i.e. the IS of inverse piezoelectrical effect). Eq. (10) thus is transformed into the equation for the IS of EIPR

$$r_{k\ell}^{\prime 0}(\theta,\varphi) = r_{i\ell}^{\prime}(\theta,\varphi)n_{i}^{\prime 3}(\theta,\varphi) - r_{j\ell}^{\prime}(\theta,\varphi)n_{j}^{\prime 3}(\theta,\varphi) - 2[n_{i}^{\prime}(\theta,\varphi) - n_{j}^{\prime}(\theta,\varphi)]d_{\ell k}^{\prime}(\theta,\varphi).$$
(17)

Since there are four different types of the IS describing the liner EO effect, Eq. (11) can be then split into the three IS of EIPR for the following geometries:

•
$$\mathbf{r} \| \mathbf{i} \| \boldsymbol{\ell} \| X'_3, \mathbf{j} \| X'_2, \mathbf{k} \| X'_1$$
:

$$r_{k\ell}^{o(\ell)}(\theta,\varphi) = r_{ii}^{\prime}(\theta,\varphi)n_{i}^{\prime3}(\theta,\varphi) - r_{j\ell}^{\prime(\ell)}(\theta,\varphi)n_{j}^{\prime3}(\theta,\varphi) - 2[n_{i}^{\prime}(\theta,\varphi) - n_{j}^{\prime}(\theta,\varphi)]d_{\ell \ell}^{\prime (\ell)}(\theta,\varphi),$$
(18)

•
$$\mathbf{r} \| \mathbf{i} \| X'_3, \mathbf{j} \| \boldsymbol{\ell} \| X'_2, \mathbf{k} \| X'_1$$
:

$$r_{k\ell}^{o(i)}(\theta,\varphi) = r_{i\ell}^{(i)}(\theta,\varphi) n_{i}^{'3}(\theta,\varphi) - r_{jj}^{'}(\theta,\varphi) n_{j}^{'3}(\theta,\varphi) - 2[n_{i}^{'}(\theta,\varphi) - n_{j}^{'}(\theta,\varphi)] d_{\ell k}^{'(i)}(\theta,\varphi),$$
(19)

• $\mathbf{r} \| \mathbf{k} \| \boldsymbol{\ell} \| X'_3, \, \mathbf{j} \| X'_2, \, \mathbf{i} \| X'_1)$:

$$r_{k\ell}^{o(k)}(\theta,\varphi) = r_{i\ell}^{\prime(k)}(\theta,\varphi) n_{i}^{\prime3}(\theta,\varphi) - r_{j\ell}^{\prime(\ell)}(\theta,\varphi) n_{j}^{\prime3}(\theta,\varphi) - 2[n_{i}^{\prime}(\theta,\varphi) - n_{j}^{\prime}(\theta,\varphi)] d_{\ell\ell}^{\prime(k)}(\theta,\varphi).$$
(20)

Here, the superscript index in brackets indicates the direction being parallel to the radius-vector of corresponding IS. In the case of the point group symmetry 3m, the IS $r'_{ii}(\theta,\varphi)$, $r'_{il}^{(\ell)}(\theta,\varphi)$, $r'_{il}^{(\ell)}(\theta,\varphi)$ and $r'_{il}^{(k)}(\theta,\varphi)$ are given by Eqs. (7)–(10). Accordingly, $r'_{jj}(\theta,\varphi)\equiv r'_{jj}(\varphi)=r'_{ii}(\theta=90^\circ; \varphi \rightarrow \varphi+90^\circ)=r_{22}\cos 3\varphi$. The spatial anisotropy of the inverse piezoelectrical effect reads as

$$d_{\ell k}^{\prime (\ell)}(\theta, \varphi) = -d_{22} \cos^2 \theta \sin \theta \sin 3\varphi + d_{31} \cos^3 \theta + (d_{33} - d_{15}) \sin^2 \theta \cos \theta,$$
(21)

$$d_{\ell k}^{\prime (1)}(\theta, \varphi) = -d_{22}\cos^2\theta\,\cos\,3\varphi,\tag{22}$$

$$d_{\ell\ell}^{\prime(k)}(\theta,\varphi) = -d_{22}\sin^{3}\theta\sin^{3}\varphi + d_{33}\cos^{3}\theta + (d_{31}+d_{15})\sin^{2}\theta\cos\theta,$$
(23)

where $\mathbf{r} \|\boldsymbol{\ell}\| X'_3$, $\mathbf{k} \| X'_1$ for the IS $d'_{\ell k}{}^{(\ell)}(\theta, \varphi)$, $\mathbf{r} \| X'_3$, $\boldsymbol{\ell} \| X'_2$, $\mathbf{k} \| X'_1$ for $d'_{\ell k}{}^{(i)}(\theta, \varphi)$ and $\mathbf{r} \| \boldsymbol{\ell} \| \mathbf{k} \| X'_3$ for $d'_{\ell \ell}{}^{(k)}(\theta, \varphi)$. The spatial anisotropy of the refractive indices follows from the optical indicatrix equation, which, for the uniaxial crystals, takes the form [14]

$$n'_{i}(\theta,\varphi) = (n_{o}^{-2}\sin^{2}\theta + n_{e}^{-2}\cos^{2}\theta)^{-1/2}, \ n'_{j}(\theta,\varphi) = n_{o}.$$
 (24)

By inserting Eqs. (7)–(10) and (21)–(24) into (18)–(20) together with the corresponding magnitudes of the mechanically "unclamped" linear EO tensor components r_{ij}^{u} , the inverse piezoelectric coefficients d_{ij} and the ordinary and extraordinary refractive indices n_0 and n_e (see Ref. [8]), one obtains the IS of EIPR as presented in Figs. 4 and 3(e-g) for the pure and MgO-doped LiNbO3 crystals, respectively. Taking into account the similarity of the IS of EIPR for the MgO-doped and pure lithium niobate, we present for LiNbO3:MgO crystals only their stereographic projections (see Fig. 3(e-g)). The extreme values that correspond to each of these IS (i.e. the absolute minima and maxima) are listed in Table 2. One can see that the IS of the EIPR for the presented geometries has the same symmetry properties as the IS of the EO effect (see Figs. 2 and 3(a-d)). In particular, both IS are described by the point group symmetry 3m in accordance with Neumann's principle. The analysis shows that the piezoelectric deformation and refractive index anisotropy have rather weak influence on their shape. The maximal magnitudes of the EIPR $|r'_{k\ell}{}^{o(\ell)}(\theta_m, \phi_m)| = 432 \times 10^{-12} \text{ m/V} \text{ and } |r'_{k\ell}{}^{o(\ell)}(\theta_m, \phi_m)| =$ $475\times10^{-12}\,\text{m/V}$ as for LiNbO3 and LiNbO3:MgO, respectively, appears for both crystals in the longitudinal geometry along the same crystallographic direction, which is defined by the angles A.S. Andrushchak et al. / Optics and Lasers in Engineering 47 (2009) 24-30



Fig. 4. The IS of EIPR and their stereographic projections in LiNbO₃ crystals: (a) IS of EIPR $r'_{k\ell}^{o(\ell)}(\theta, \varphi)$ [\mathbf{r} || \mathbf{i} || \mathcal{U} | X'_3 , \mathbf{j} || \mathcal{U} || X'_3 , \mathbf{j} || X'_3 ,



Fig. 5. Examples of two different geometries of the EO cells made of LiNbO₃ (LiNbO₃:MgO) crystals. The faces coloured in gray would correspond to deposited electrodes. **E** and **k** correspond to the directions of applied electric field and light propagation, respectively. (a) The cell used in most standard applications and (b) the cell in the best-optimized geometry for electro-induced phase retardation (EIPR). The modulation efficiency of cell (b) is almost three times better compared to the modulation efficiency of the standard cell (a). The axes X_1 , X_2 and X_3 correspond to the crystallophysical coordinate system.

 $\theta_{\rm m} = 54^{\circ} (\varphi_{\rm m} = 90^{\circ}, 210^{\circ} \text{ or } 330^{\circ}) \text{ or } \theta_{\rm m} = 126^{\circ} (\varphi_{\rm m} = 30^{\circ}, 150^{\circ} \text{ or } 270^{\circ})$ (see Table 2). These magnitudes almost three times exceed the EIPR along the optical axis ($\mathbf{k} \parallel X'_3$, $\boldsymbol{\ell} \parallel X'_2$, then $\theta = 90^{\circ}$, $\varphi = 90^{\circ}$): $|r_{32}^{\circ(\ell)}| = 162 \times 10^{-12} \text{ m/V}$ or $|r_{32}^{\circ(\ell)}| = 178 \times 10^{-12} \text{ m/V}$ [1] as for LiNbO₃ or LiNbO₃:MgO, respectively. The corresponding cell geometry until now is frequently used in many standard applications (see Fig. 5(a)). Just by switching to the cell geometry as is shown in Fig. 5(b), one can improve the modulation efficiency almost three times. This results in a corresponding reduction of

driving voltages of the EO modulators or deflectors, which evidently is of great practical importance for many applications. It must be emphasized that even pure LiNbO₃ single crystals are usually characterized by intrinsic growth defects, leading to substantial changes of the EO coefficients [15]. Accordingly, depending on a crystal growth technology it causes modifications of the IS of EO or EIPR; thus optimized geometries of the EO cells should be determined for each particularly used crystal material.

4. Conclusions

We have presented here the results of spatial anisotropy of the linear EO effect in crystal materials. The method is based on the construction and analysis of the IS describing the spatial anisotropy of the longitudinal and transverse EO effects. The corresponding equations have been derived and applied to the calculations of the IS in pure and MgO-doped LiNbO₃ crystals. By means of the IS, the spatial anisotropy of the EO coupling is evaluated in these crystals. The best-optimized cell geometries suitable for the EO applications were determined within a computer simulation-based optimization procedure. The maximum magnitudes of the linear EO effect are revealed in the longitudinal and transverse geometries of both crystals along the directions, which substantially deviate from the principal crystal-lographic axes. This defines a set of the optimized sample geometries recommended for designers dealing with EO mod-

ulators or deflectors. Just by switching from the standard cell geometry to the optimized cell geometry, as determined in the present work, one may improve almost three times the modulation efficiency of EO devices based on pure and MgO-doped lithium niobate crystals.

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