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Complete sets of elastic constants and photoelastic coefficients of pure and MgO-doped lithium niobate crystals at room temperature

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This paper presents the results of ultrasonic measurements of $LiNbO_3$ and $LiNbO_3:MgO$ crystals. The tensors of piezoelectric coefficients, elastic stiffness constants, and elastic compliances are determined for both crystals at room temperature. Combining these data with the results of piezo-optical measurements, a complete set of photoelastic tensor coefficients is also calculated. Doping of $LiNbO_3$ crystals by MgO does not lead to a considerable modification of their elastic and photoelastic properties. However, $LiNbO_3:MgO$ is characterized by a considerably higher resistance with respect to powerful light radiation, making it promising for future application in acousto-optic devices that deal with superpowerful laser radiation. Presented here are the complete tensor sets of elastic constants and photoelastic coefficients of $LiNbO_3$ and $LiNbO_3:MgO$ crystals that may be used for a geometry optimization of acousto-optical interaction providing the best diffraction efficiency of acousto-optical cells made of these materials. © *2009 American Institute of Physics*. [doi:10.1063/1.3238507]

I. INTRODUCTION

Photoelasticity has considerable practical importance and is applicable in many solid-state electronic and optoelectronic devices such as acousto-optical modulators or deflectors, photoelastic modulators, and piezo-optic sensors. A growing interest in these studies is due to recent developments in fields of crystal growth and technology,[1–3] leading to the appearance of new efficient electro-optic and/or photoelastic materials.[4–9] The application of photoelastic crystals, usually as media for acousto-optic modulators or deflectors, requires knowledge regarding the spatial anisotropy of their optical and acoustical properties. The main problems arise here with respect to a geometry optimization of acousto-optic or photoelastic interactions, which makes it only possible to use these materials with maximal diffraction or photoelastic efficiency. Such problems are frequently solved in terms of indicative surfaces, which may be calculated if only a complete set of elastic and photoelastic tensor coefficients is known. The elastic properties of crystal materials are usually studied by ultrasonic methods representing the most reliable and accurate techniques in this respect. The photoelastic coefficients can be determined by acousto-optic methods, such as the Dixon–Cohen technique.[10] However, these methods are rather ambiguous in the determination of the sign of photoelastic coefficients, especially in the case of low-symmetry crystals. Alternatively, a complete set of photoelastic coefficients can also be obtained via the piezo-optic measurements representing in this sense the most appropriate technique that is suitable for a precise and unambiguous determination of both their absolute magnitudes and signs.[9]

In this paper, we report a complete set of the piezoelectric coefficients, elastic constants, and photoelastic tensor coefficients of MgO-doped (7% in melt composition) $LiNbO_3$ crystals at room temperature. Our interest in such materials is basically due to their high resistance with respect to power laser radiation,[11] which appears to be very important for many applications.[12] The photoelastic coefficients of MgO-doped $LiNbO_3$ crystals (hereafter also referred to as $LiNbO_3:MgO$) are obtained here using the results of the ultrasonic and piezo-optic measurements. Accordingly, the methodology presented here can be considered as a guide suitable for the determination of the photoelastic parameters in similar crystal materials as well as crystals of any other symmetry. It has also been applied to pure $LiNbO_3$; thus, the photoelastic characteristics determined for these crystals are compared with the ones obtained by other authors. This allows us to verify the experimental and calculation aspects of the methodology developed in actual work.

II. ELASTIC CONSTANTS AND PIEZOELECTRIC COEFFICIENTS OF $LINBO_3$ AND $LINBO_3:MGO$ CRYSTALS

The elastic and piezoelectric properties of pure $LiNbO_3$ have been repeatedly studied by many authors (see, e.g.,

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TABLE I. Measured sound velocities in pure $LiNbO_3$ (smp-1, smp-2, and smp-3) and $LiNbO_3$:MgO. L, longitudinal mode; T, transverse mode; QL, quasilongitudinal mode; QT, quasitransverse mode; $\Delta V_{i,j}$, difference between the measured sound velocities in $LiNbO_3$:MgO and $LiNbO_3$ (smp-1).

			Sound velocity (m/s)					
Polarization direction	Propagation direction	Acoustic mode	$LiNbO_3$ smp-1	$LiNbO_3$ smp-2	$LiNbO_3$ smp-3	$LiNbO_3$:MgO	$\Delta V_{i,j}$	Ref. 16
[100]	[100]	L	6560	6550	6572	6654	94	⋯
[010]	[100]	T	4040	4029	4038	4069	29	4050
[001]	[100]	T	4758	4735	4752	4846	88	4800
[001]	[001]	L	7351	7329	7365	7399	48	7330
[010]	[001]	T	3598	3595	3601	3627	29	3570
[010]	[010]	QL	6832	6842	6842	6894	62	6880
[001]	[010]	QT	4463	4449	4453	4543	80	4490
[100]	[010]	T	3951	3942	3949	3997	46	3960
[100]	[011]	T	3998	3991	3996	4032	34	3960
$[01\bar{1}]$	[011]	QT	4003	4001	4009	4039	36	4030
[011]	[011]	QL	7366	7356	7376	7426	60	7380
[100]	$[01\bar{1}]$	T	3544	3540	3549	3584	40	3570
[011]	$[01\bar{1}]$	QT	4126	4121	4125	4158	32	4200
$[01\bar{1}]$	$[01\bar{1}]$	QL	7086	7076	7096	7146	60	7010

Refs. 13–17); however, the magnitudes of the same elastic constants or piezoelectric coefficients obtained in these works in many cases considerably deviate each other. Such discrepancies in principle may have a number of reasons, one of which could be the different or sometimes even uncontrolled crystal quality of materials used in these studies. This fact has stimulated us to perform again the elastic measurements of pure $LiNbO_3$ on the samples supplied by three different companies, namely, Scientific Research Co. (SRC) "Carat" (Lviv, Ukraine), Bogorodeck Chemical Co. (Bogorodeck, Russia), and Yamatsu Corporation (Japan) called hereafter as smp-1, smp-2, and smp-3, respectively. $LiNbO_3$:MgO crystals (7% of MgO in melt composition) has been synthesized by SRC "Carat." Importantly, a required set of acoustic, piezo-optic, dielectric, and other measurements has been performed in each case on the same samples that have been previously very carefully selected by means of polarization microscope in order to choose the ones of best quality being free from growth strains, microcracks, or other defects.

The crystals $LiNbO_3$:MgO, like pure $LiNbO_3$, are characterized by the point group symmetry $3m$. The tensor constants of physical properties are usually referred to an orthonormal set of axes, $X=1$, $Y=2$, and $Z=3$, which in the case of the trigonal structure $3m$ possess a standard orientation where its Z-axis is chosen parallel to the optical axis and coincides with a threefold axis. Accordingly, in the principal crystallophysical coordinate system, $LiNbO_3$ and $LiNbO_3$:MgO crystals are characterized by six independent elastic stiffness constants, four independent piezoelectric coefficients, and two independent dielectric constants, as shown below,[16]

$$\begin{matrix} C_{11} & C_{12} & C_{13} & C_{14} & 0 & 0 & 0 & -e_{22} & e_{31} \\ C_{12} & C_{11} & C_{13} & -C_{14} & 0 & 0 & 0 & e_{22} & e_{31} \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 & 0 & 0 & e_{33} \\ C_{14} & -C_{14} & 0 & C_{44} & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & 0 & C_{44} & C_{14} & e_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{14} & C_{66} & -e_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{15} & -e_{22} & \varepsilon_{11} & 0 & 0 \\ -e_{22} & e_{22} & 0 & e_{15} & 0 & 0 & 0 & \varepsilon_{11} & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 & 0 & 0 & \varepsilon_{33} \end{matrix},$$

where C_{ilkm} are the elastic stiffness constants presented in the matrix form $[C_{\lambda\mu}=C_{ilkm},\ il\leftrightarrow\lambda=1,\ldots,6,\ km\leftrightarrow\mu=1,\ldots,6$ (Ref. 18)], e_{lmi} and e_{pqk} are the piezoelectric tensor coefficients presented in the matrix form $[e_{l\lambda}=e_{lmi},\ mi\leftrightarrow\lambda=1,\ldots,6$ (Ref. 18)], and ε_{rs} are the dielectric tensor constants. We report first a complete determination of the elastic constant matrix via the measurements of the ultrasonic phase velocities along different crystallographic directions. Direct and simple relations between measured velocities and elastic constants are possible only for the four stiffness constants, namely, C_{11}, C_{12}, C_{44}, and C_{66}. The remaining two constants C_{33} and C_{13} occur, coupled together with piezoelectric coefficients in more complicated relationships. The determination of all six elastic stiffness constants of the trigonal system is therefore demanding, both experimentally and computationally, and requires measurements of the velocities of the longitudinal (L), quasilongitudinal (QL), transverse (T), or quasitransverse (QT) ultrasonic waves in at least six different directions: [100], [010], [001], [110], [101], and [011]. Table I gives the details regarding the propagation direction and polarization of ultrasonic waves used in the study. For the piezoelectric crystals, the relationships between measured velocities, V, the elastic stiffness constants, C_{ilkm}, and piezoelectric coefficients, e_{lmi} and e_{pqk}, follow from the Christoffel equation:

$$\left| \rho V^2 \delta_{ik} - C_{ilkm} n_l n_m - \frac{(e_{lmi} n_l n_m)(e_{pqk} n_p n_q)}{\varepsilon_0 \varepsilon_{rs} n_r n_s} \right| = 0, \tag{1}$$

where ρ is the crystal density, n_l, n_m, n_p, n_q, n_r, and n_s are the components of the unit vector $\mathbf{n}$ coinciding with the direction of sound propagation, δ_{ik} is the Kronecker delta function, and $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m is the vacuum permittivity. In the case of point group $3m$, Eq. (1) is translated to the following relations:[14]

$$C_{11} = \rho V_{1,1}^2, \quad C_{14} = \frac{\rho(V_{1,4}^2 - V_{1,\bar{4}}^2)}{2}, \tag{2}$$

$$C_{44} = \rho V_{1,3}^2, \quad C_{66} = \rho V_{1,2}^2, \quad C_{12} = C_{11} - 2C_{66}, \tag{3}$$

$$C_{33} = \rho V_{3,3}^2 - \frac{e_{33}^2}{\varepsilon_0 \varepsilon_{33}}, \tag{4}$$

$$C_{13} = 2\left[\sqrt{\beta} - \left\{ \frac{(e_{22} + e_{15} + e_{31})(e_{15} + e_{33})}{2\varepsilon_0(\varepsilon_{22} + \varepsilon_{33})} \right\} \right] - C_{14} + C_{44}, \tag{5}$$

where $\beta = \alpha_{33}\alpha_{22} - (\rho V_{\bar{4},4} V_{4,4})^2$ $[\alpha_{33} = (C_{33} + C_{44})/2 + (e_{15} + e_{33})^2/2\varepsilon_0(\varepsilon_{22} + \varepsilon_{33})$ and $\alpha_{22} = (C_{11} - 2C_{14} + C_{44})/2 + (e_{22} + e_{15} + e_{31})^2/2\varepsilon_0(\varepsilon_{22} + \varepsilon_{33})]$ and

$$C_{13} = 2\left[-\sqrt{\gamma} - \left\{ \frac{(e_{22} - e_{15} - e_{31})(e_{15} + e_{33})}{2\varepsilon_0(\varepsilon_{22} + \varepsilon_{33})} \right\} \right] - C_{14} - C_{44}, \tag{6}$$

where $\gamma = \alpha_{33}\alpha'_{22} - (\rho V_{4,\bar{4}} V_{\bar{4},\bar{4}})^2$ $[\alpha'_{22} = (C_{11} + 2C_{14} + C_{44})/2 + (e_{22} - e_{15} - e_{31})^2/2\varepsilon_0(\varepsilon_{22} + \varepsilon_{33})]$,

$$e_{15}^2 = \varepsilon_0 \varepsilon_{11} \frac{\rho^2(V_{2,1}^2 V_{3,1}^2 - V_{2,2}^2 V_{3,3}^2) - C_{44}(C_{66} - C_{11})}{C_{66} - C_{11}}, \tag{7}$$

$$e_{22}^2 = \varepsilon_0 \varepsilon_{22} \left[\rho(V_{2,2}^2 + V_{3,2}^2) - \left(C_{11} + C_{44} + \frac{e_{15}^2}{\varepsilon_0 \varepsilon_{22}} \right) \right], \tag{8}$$

$$e_{31} = \frac{\varepsilon_0(\varepsilon_{22} + \varepsilon_{33})[\rho(V_{\bar{4},4}^2 + V_{4,4}^2) - \rho(V_{4,\bar{4}}^2 + V_{\bar{4},\bar{4}}^2) + 2C_{14}]}{2e_{22}} - e_{15}, \tag{9}$$

$$e_{33} = e_{15}\frac{\varepsilon_{33}}{\varepsilon_{22}} - \sqrt{\frac{\varepsilon_{33}}{\varepsilon_{22}} \left[e_{15}^2 \left(1 + \frac{\varepsilon_{33}}{\varepsilon_{22}} \right) - \varepsilon_0 [2\rho(V_{\bar{4},4}^2 + V_{4,4}^2) - 2\alpha - \rho V_{3,3}^2 - C_{44}](\varepsilon_{22} + \varepsilon_{33}) \right]}, \tag{10}$$

where $\alpha = \frac{1}{2}(C_{11} - 2C_{14} + C_{44}) + (e_{22} + e_{15} + e_{31})^2/2\varepsilon_0(\varepsilon_{22} + \varepsilon_{33})$.

Here, $V_{i,j}$ are the sound velocities where the indices i and $j = 1$, 2, 3, 4 or $\bar{4}$ denote the wave polarization and propagation directions, respectively, the index with value 4 defines the diagonal, i.e., 45° direction between the positive directions (according to Institute of Radio and Engineering (IRE) standards[19]) of the axes X_2 and X_3, whereas the index with value $\bar{4}$ defines the direction perpendicular to the direction 4. Further determination of elastic and piezoelectric constants is based on measured sound velocities. A set of ultrasonic geometries required for a complete determination of the elastic and piezoelectric tensors is given in Table I. The sequence of the calculations is the following. One should determine first the elastic constants C_{11}, C_{14}, C_{44}, and C_{66} by means of Eqs. (2) and (3); then, the piezoelectric constants e_{15}, e_{22}, e_{31}, and e_{33} are calculated using Eqs. (7)–(10), and finally the elastic stiffness constants C_{33} and C_{13} are determined by means of Eqs. (4)–(6).

The velocities of the L, QL, T, and QT ultrasonic waves were measured by the pulse-echo overlap method (Papadakis[20]) with an absolute accuracy of about 0.1%. The longitudinal and transverse acoustic waves in the samples were excited by $LiNbO_3$ transducers (resonance frequency $f = 10$ MHz, bandwidth $\Delta f = 0.1$ MHz, and acoustic pulse power $P_a = 1-2$ W). The results of acoustic measurements in $LiNbO_3$ (smp-1, smp-2, and smp-3) and $LiNbO_3{:}MgO$ crystals are given in Table I. One can realize that the velocity magnitudes determined for different samples of pure $LiNbO_3$ crystals only insignificantly differ with each other and in most cases remain within the experimental error. Alternatively, small differences in sound velocities can also be attributed to eventually different technologies of crystal growth. By comparing the results for pure and MgO-doped $LiNbO_3$ crystals, some tendencies must be admitted; in particular, sound velocities are larger for $LiNbO_3{:}MgO$ crystals. The corresponding difference $\Delta V_{i,j} = V_{i,j}(LiNbO_3{:}MgO) - V_{i,j}(LiNbO_3, smp-1)$ appears in the range of 29–94 m/s, i.e., considerably exceeds the magnitude of an experimental error.

The elastic stiffness constants and piezoelectric coefficients have been determined by means of Eqs. (2)–(10) using measured magnitudes of sound velocities $V_{i,j}$ (Table I), measured crystal densities $\rho = 4.628 \times 10^3$ kg/m^3 ($LiNbO_3$) and $\rho = 4.638 \times 10^3$ kg/m^3 ($LiNbO_3{:}MgO$), and measured dielectric constants $\varepsilon_{11} = \varepsilon_{22} = 44$ and $\varepsilon_{33} = 27.9$, which for both crystals have been found at the same magnitudes within the experimental error. Accordingly, Table II presents complete sets of determined elastic stiffness constants ($C_{\lambda\mu}$), piezoelectric coefficients ($e_{l\lambda}$), piezoelectric moduli ($d_{l\mu} = e_{l\lambda} S_{\lambda\mu}$), and elastic compliances ($S_{\lambda\mu} = C_{\lambda\mu}^{-1}$). The errors have been

TABLE II. Calculated piezoelectric coefficients $e_{l\lambda}$, piezoelectric moduli $d_{l\mu}$, elastic stiffness constants $C_{\lambda\mu}$, and elastic compliances $S_{\lambda\mu}$ of pure and MgO-doped lithium niobate crystals.

	Results of our measurements				Literature data for $LiNbO_3$				
	$LiNbO_3$ (smp-1)	$LiNbO_3$ (smp-2)	$LiNbO_3$ (smp-3)	$LiNbO_3$:MgO	Ref. 13	Ref. 14	Ref. 15	Ref. 16	Ref. 17
				Piezoelectric constants (C/m^2)					
e_{15}	3.67 ± 0.03	3.71 ± 0.03	3.63 ± 0.03	3.75 ± 0.03	3.61	3.60	3.7	3.76	3.8
e_{22}	2.38 ± 0.06	2.38 ± 0.06	2.38 ± 0.06	2.32 ± 0.06	2.40	2.52	2.5	2.43	2.5
e_{31}	0.34 ± 0.14	0.29 ± 0.14	0.37 ± 0.14	0.40 ± 0.15	0.28	0.75	0.2	0.23	⋯
e_{33}	1.6 ± 0.9	1.6 ± 0.9	1.7 ± 1.1	1.1 ± 0.6	1.59	1.67	1.3	1.33	1.7
				Piezoelectric moduli (10^{-12} C/N)					
d_{15}	66.6 ± 0.5	67.9 ± 0.5	65.8 ± 0.5	66.6 ± 0.5	65.36	64.3	68	69.2	78
d_{22}	20.1 ± 1.3	19.8 ± 1.3	20.1 ± 1.3	19.2 ± 1.2	20.29	20.6	21	20.8	19.2
d_{31}	−0.57 ± 1.85	0.44 ± 1.50	−0.57 ± 2.20	0.40 ± 1.34	−1.22	1.15	−1	−0.85	1.3
d_{33}	6.9 ± 4.5	4.2 ± 2.8	7.3 ± 5.3	4.1 ± 2.7	8.27	6.53	6	6.0	18.9
				Elastic constants (GPa)					
C_{11}	199.2 ± 0.4	198.6 ± 0.4	199.9 ± 0.4	205.4 ± 0.4	199	202	203	203.0	199
C_{12}	54.7 ± 0.5	54.7 ± 0.5	55.6 ± 0.5	57.2 ± 0.5	53.8	55.7	53	57.3	⋯
C_{13}	70 ± 19	69 ± 20	70 ± 23	74 ± 11	71.4	69	75	75.2	74.2
C_{14}	7.9 ± 0.1	7.8 ± 0.1	7.8 ± 0.1	7.9 ± 0.1	7.85	7.48	9	8.5	7.7
C_{33}	240 ± 11	238 ± 12	240 ± 14	249 ± 5	237.2	240	245	242.4	238
C_{44}	59.9 ± 0.1	59.8 ± 0.1	60.0 ± 0.1	60.9 ± 0.1	⋯	60.7	60	59.5	⋯
C_{66}	72.2 ± 0.1	71.9 ± 0.1	72.2 ± 0.1	74.1 ± 0.1	⋯	72.9	75	72.8	⋯
				Elastic compliances $(GPa)^{-1}$					
S_{11}	5.86 ± 0.38	5.87 ± 0.36	5.84 ± 0.38	5.71 ± 0.35	5.86	5.77	5.78	5.831	5.81
S_{12}	−1.16 ± 0.38	−1.18 ± 0.36	−1.18 ± 0.38	−1.13 ± 0.35	−1.12	−1.77	−1.01	−1.150	⋯
S_{13}	−1.37 ± 0.71	−1.37 ± 0.41	−1.37 ± 0.86	−1.36 ± 0.41	−1.43	−1.32	−1.47	−1.452	1.5
S_{14}	−0.93 ± 0.02	−0.93 ± 0.02	−0.91 ± 0.02	−0.89 ± 0.02	−0.91	−0.85	−1.02	−1.000	0.91
S_{33}	4.97 ± 0.37	5.00 ± 0.74	4.98 ± 0.48	4.82 ± 0.16	5.08	4.92	5.02	5.026	5.12
S_{44}	16.94 ± 0.03	16.96 ± 0.04	16.90 ± 0.03	16.65 ± 0.03	16.88	16.6	17.0	17.10	17.6
S_{66}	14.04 ± 0.02	14.10 ± 0.3	14.13 ± 0.02	13.68 ± 0.02	13.97	13.9	13.6	13.96	14

calculated in each case as mean square values taken over all errors of measured magnitudes that enter into Eqs. (2)–(10). For this reason, the error value is considerably smaller when corresponding magnitudes are calculated directly from the measured sound velocities only, which is the case of, e.g., C_{11}, C_{12}, C_{44}, and C_{66} elastic constants. The worst accuracy takes place when the magnitude of the elastic constant C_{13} is determined since one deals in this case with rather complicated relationships (5) and (6), which couples to the velocities together with a large number of piezoelectric coefficients. The sign of the elastic constant C_{14} has been determined according to the rule[19] defining the positive directions for X_2 and X_3 axes. Comparing the magnitudes of corresponding tensor constants that have been determined for smp-1, smp-2, and smp-3 of pure $LiNbO_3$, one can find a satisfactory agreement, usually within the experimental errors, which indeed verifies the used experimental technique and the developed methodology.

III. PHOTOELASTIC TENSOR COEFFICIENTS OF $LINBO_3$ AND $LINBO_3$:MGO CRYSTALS

Photoelasticity is usually explained in terms of the optical anisotropy induced by a lattice distortion and plays a key role in mechanisms of acousto-optic light diffraction or photoelastic light modulation. In a classical meaning, the photoelastic effect describes the change in the optical polarization tensor constants $a_i \equiv a_{lk}$ caused by internal mechanical strains $u_n \equiv u_{jr}$, which in the linear regime is expressed as

$$\Delta a_i = p_{in} u_n, \qquad (11)$$

where p_{in} is the tensor of photoelastic coefficients presented in the matrix form $[p_{in} \equiv p_{lkjr}, lk \leftrightarrow i = 1, \ldots, 6, jr \leftrightarrow n = 1, \ldots, 6$ (Ref. 18)]. Following Eq. (11), the photoelastic coefficients p_{in} could be, in principle, directly determined by measuring a strain induced optical refraction or birefringence, which in practice is not so easy or even impossible

TABLE III. Piezo-optic constants π_{im} of $LiNbO_3$:MgO and $LiNbO_3$ crystals (Ref. 21).

π_{im}, Br	π_{11}	π_{12}	π_{13}	π_{31}	π_{33}	π_{14}	π_{41}	π_{44}
$LiNbO_3$:MgO	−0.43	+0.15	+0.78	+0.50	+0.32	−0.80	−0.90	+2.0
$LiNbO_3$	−0.38	+0.09	+0.80	+0.50	+0.20	−0.81	−0.88	+2.25

TABLE IV. Photoelastic constants p_{im} of $LiNbO_3$:MgO and $LiNbO_3$ crystals.

	$LiNbO_3$	$LiNbO_3$:MgO	Ref. 22	Ref. 23	Ref. 24	Ref. 25[a]	Ref. 26[a]	Ref. 27[a]	Ref. 28[a]	Ref. 29	Ref. 30
p_{11}	-0.021 ± 0.018	-0.015 ± 0.013	-0.026	-0.02	0.032	0.034	0.021	0.021	0.036	0.025	···
p_{12}	0.060 ± 0.019	0.058 ± 0.014	0.09	0.08	0.063	0.072	0.096	0.088	0.072	0.079	···
p_{13}	0.172 ± 0.029	0.174 ± 0.026	0.133	0.13	0.069	0.139	0.149	0.126	0.135	0.132	0.096
p_{31}	0.141 ± 0.017	0.155 ± 0.018	0.179	0.17	0.153	0.178	0.138	0.138	0.178	0.168	···
p_{33}	0.118 ± 0.020	0.154 ± 0.019	0.071	0.07	0.061	0.06	0.078	0.069	0.066	0.068	0.093
p_{14}	-0.052 ± 0.007	-0.044 ± 0.007	-0.075	-0.08	···	0.066	0.055	0.055	0.086	0.1	0.07
p_{41}	-0.109 ± 0.017	-0.149 ± 0.032	-0.151	-0.15	0.136	0.154	···	0.12	0.155	0.158	···
p_{44}	0.121 ± 0.019	0.136 ± 0.030	0.146	0.12	···	0.3	0.019	0.152	···	···	···

[a] The results that have been recalculated in Ref. 22.

due to a problem related to a precise determination of the internal strain tensor u_n inside the crystal sample. Alternatively, the photoelastic coefficients can be determined by acousto-optical methods; however, such techniques are rather ambiguous in the determination of the sign of photoelastic coefficients. A solution of this problem lies in piezo-optical measurements in which the tensor of piezo-optic coefficients $\pi_{im} \equiv \pi_{lkjg}$ $[lk \leftrightarrow i=1,\ldots,6, jr \leftrightarrow n=1,\ldots,6$ (Ref. 18)] is determined via measuring a stress induced changes in the refraction indices Δn_i and then calculated as $\pi_{im}=\Delta(n_i^{-2})/\sigma_m = -2n_i^{-3}\Delta n_i/\sigma_m$, where σ_m is the stress applied to the sample. Such measurements may be quite accurate, taking into account a well defined magnitude of applied σ_m and precise interferometric techniques that are usually used to measure n_i and Δn_i. The photoelastic coefficients are expressed then through the tensors of piezo-optic coefficients as

$$p_{in} = \pi_{im} C_{mn} \tag{12}$$

and can be easily determined if a complete set of the elastic tensor constants C_{mn} is known.

A complete set of piezo-optic tensor coefficients of pure and MgO-doped $LiNbO_3$ crystals has been reported in our recent work.[21] The piezo-optic measurements were performed exactly on the same samples (smp-1, SRC "Carat"), i.e., on those ones for which the elastic tensor constants have been measured later (see Table II). The magnitudes of the piezo-optic coefficients are given in Table III. Table IV presents in the first two columns the photoelastic tensor coefficients of $LiNbO_3$ and $LiNbO_3$:MgO crystals, as calculated by means of Eq. (12), and the data presented in Tables II and III. For comparison, it lists also the magnitudes of photoelastic coefficients of pure $LiNbO_3$ crystals (see columns marked by Refs. 22–30) obtained by other authors, preferably applying the Dixon–Cohen technique. The accuracies accompanying the calculated magnitudes of photoealastic coefficients (see the first two columns of Table IV) represent mean square errors being evaluated as

$$\delta p_{in} = [(\delta\pi_{im})^2(C_{mn})^2 + (\pi_{im})^2(\delta C_{mn})^2]^{1/2}, \tag{13}$$

where $\delta\pi_{im}$ and δC_m are the absolute errors taken from Tables II and III, respectively. Most of the photoelastic coefficients of pure $LiNbO_3$, which have been evaluated from measured piezo-optic coefficients and elastic constants, within the experimental error coincide with the results obtained by the Dixon–Cohen method in Refs. 22–30. It is also important that the photoelastic coefficients p_{11}, p_{14}, and p_{41} exhibit negative values consistent with those in Refs. 22 and 30. Such comparison indeed verifies a validity of the techniques offered in the present work. Doping of $LiNbO_3$ crystals by MgO does not lead to a considerable modification of their photoelastic properties. From Table IV, it follows that the magnitudes of corresponding photoelastic coefficients p_{km} of pure and MgO-doped $LiNbO_3$ crystals are close to each other, and their deviations appear in most cases within the experimental error.

IV. CONCLUSIONS

In conclusion, we present here the results of ultrasonic measurements of $LiNbO_3$ and $LiNbO_3$:MgO crystals. The tensors of piezoelectric coefficients, piezoelectric moduli, elastic stiffness constants, and elastic compliances are determined for both crystals at room temperature. Combining these data with the results obtained in previous piezo-optical studies,[21] we have also calculated a complete set of the photoelastic coefficients in these materials. Doping of $LiNbO_3$ crystals by MgO does not lead to a considerable modification of their elastic and photoelastic properties. However, $LiNbO_3$:MgO is characterized by about four times higher resistance with respect to powerful light radiation,[11] making it promising for future application in acousto-optic devices such as modulators and deflectors that deal with superpowerful laser radiation. Presented here are the complete sets of elastic constants and photoelastic tensor coefficients of $LiNbO_3$ and $LiNbO_3$:MgO crystals that may be used for geometry optimization of acousto-optical interaction providing the best diffraction efficiency of acousto-optical cells made of these materials. A spatial related analysis of the acousto-optic diffraction efficiency [figure of merit $M_2(\theta,\varphi)$] for both crystals will be published elsewhere soon.

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