Anisotropy of piezo-, elasto- and acousto-optic properties of $La_3Ga_5SiO_{14}$ crystal

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ABSTRACT

All piezo-optical and elasto-optical tensor constants of $La_3Ga_5SiO_{14}$ crystals at room temperature are obtained. On the basis of filled matrix of π_{im} and p_{in} the indicative surfaces for longitudinal or transverse piezo- and elasto-optical effects were constructed, and their extreme values were determined. The largest value of the figure of merit $M_2 = 72.3\times10^{-15}$ s^3/kg for $La_3Ga_5SiO_{14}$crystals are presented.

Keywords: indicative surfaces, piezo - and electro-optical coefficients, figure of merit, $La_3Ga_5SiO_{14}$ crystals.

1. CALCULATION OF THE PIEZO- AND ELASTO-OPTIC TENSOR COMPONENTS.

Crystals of langasite ($La_3Ga_5SiO_{14}$), which first grown a quarter of a century ago, was turned out as piezoelectric and electromechanic material [1]. Optical properties were studied in [2].

The main purpose of this work was evaluation of acousto-optic properties of langasite crystals. For that the absolute piezo-optic coefficients π_{im} was measured and then the matrix of elastooptic coefficients p_{im} was calculated [3].

The results of langasite piezo-optic coefficients are shown in Table 1. The coefficient calculations π_{im} were conducted basing on the experimental measuring of induced by mechanical stress σ_m pathlength changes $\delta\Delta_k$ of optical beam passed through sample (interferometer method is applied). The main piezooptic coefficients that describe the change of the crystal refraction indexes $n_1= n_2$ and n_3 under action of mechanical stress were calculated on the base of the known relationship [4]

$$\delta\Delta_k=-0.5\pi_{im}\sigma_m d_k n_i^3+S_{km}\sigma_m d_k(n_i-1), \tag{1}$$

where n_i – crystal refraction index, d_k – sample thickness (cube edge ~ 7÷8 mm) in light propagation direction, S_{km} – elastic compliance coefficient; the **k**, **i**, **m** indexes designate the light propagation, light polarization and one-dimensional pressure action directions, respectively.

The langasite crystal has the five main non-zero independent coefficients due to high symmetry (32 class) - π_{11}, π_{12}, π_{13}, π_{31} ? π_{33}. And other three independent piezo-optical coefficients π_{im} are non-main: π_{41} (rotative), π_{14} (shear)? π_{44} (rotative-shear). The relationships to calculation of these piezooptic coefficients include the complicated sums of main and non-main piezo-optic coefficients and coefficients S_{im}. They are presented elsewhere [5] for crystals of lithium niobate, which have the same matrix of piezo-optic coefficients possessing by 3m symmetry class. It is known [5-7], that coefficients π_{14} ? π_{41} is determined with accuracy to a sign, and π_{44} – with accuracy to an absolute value, at condition that the positive signs are not appropriated to axes of crystallophysical co-ordinate system (axes of optical indicatrix). It can be made both on base of piezoelectric effect [6], and piezo-optic effect [6, 7]. For statement of axes signs we have used such criteria: $\pi_{14} > 0$ [7].

For analysis of langasite crystals acousto-optic efficiency it is necessary, firstly, to determine the elasto-optic coefficients p_{im}. Using the magnitudes of the piezo-optic tensor components p_{im} can be calculated the elasto-optic tensor constants p_{in} (see Table 1) according to well-known relation:

$$p_{in} = p_{im} C_{mn}, \tag{2}$$

where C_{mn} is the elastic constant matrix. As one can seen the $La_3Ga_5SiO_{14}$ crystals have only one large magnitudes of elasto-optic coefficients.

Table 1. The average values of piezo-optic coefficients π_{im}, elastic coefficients C_{mn} from paper [3], and calculated values of elasto-optic coefficients p_{in} for $La_3Ga_5SiO_{14}$ crystal (for 20°C temperature and $\lambda = 0.6328$ μm light wave length)

Index *im*, *mn* or *in*	p_{im}, Br=10^{-12} m^2/N	C_{mn}, GPa	p_{in}
11	-2.0	18.6	0.002
12	0.1	9.9	0.035
13	0.4	9.4	0.066
31	0.55	9.4	0.082
33	-1.3	24.1	-0.17
14	0.34	-1.5	0.021
41	0.33	-1.5	0.023
44	0.35	5.3	0.0087

2. THE INDICATIVE SURFACES

The analysis of spatial anisotropy of the piezo- and elasto-optic properties is based on the indicative surfaces. The corresponding method developed in [8] allows to describe both qualitatively and quantitatively the anisotropy of any physical effect in the crystals. We must mention that any ambiguity both in determination of piezo- or elasto-optic constants as well as then in the constructing of the indicative surfaces can be removed if to use always the same coordinate system, in particular the one, which was used for the determination of the piezo-optic coefficients. In the piezo- or elasto-optic measurements it is important also to choose properly the positive directions of principal coordinate system. In the spherical coordinate system (θ,φ) these equations for langasite crystals get the form:

$$\pi'_{ii}(\theta,\varphi)=\pi_{11}\sin^4\theta+(\pi_{13}+\pi_{31}+2\pi_{44})\sin^2\theta\cos^2\theta+\pi_{33}\cos^4\theta+(\pi_{14}+2\pi_{41})\sin^3\theta\cos\theta\sin3\varphi, \quad (3)$$

$$\pi'^{(i)}_{im}(\theta,\varphi)=\pi_{12}\sin^2\theta + \pi_{31}\cos^2\theta - 2\pi_{41}\sin\theta\cos\theta\sin3\varphi, \quad (4)$$

$$\pi'^{(m)}_{im}(\theta,\varphi)=\pi_{12}\sin^2\theta + \pi_{13}\cos^2\theta - \pi_{14}\sin\theta\cos\theta\sin3\varphi, \quad (5)$$

$$p'_{ii}(\theta,\varphi)=p_{11}\sin^4\theta+(p_{13}+p_{31}+4p_{44})\sin^2\theta\cos^2\theta+p_{33}\cos^4\theta+2(p_{14}+p_{41})\sin^3\theta\cos\theta\sin3\varphi, \quad (6)$$

$$p'^{(i)}_{in}(\theta,\varphi)= p_{12}\sin^2\theta + p_{31}\cos^2\theta - 2p_{41}\sin\theta\cos\theta\sin3\varphi, \quad (7)$$

$$p'^{(n)}_{in}(\theta,\varphi)= p_{12}\sin^2\theta + p_{13}\cos^2\theta - 2p_{14}\sin\theta\cos\theta\sin3\varphi, \quad (8)$$

where $\pi'_{ii}(\theta,\varphi)$, $p'_{ii}(\theta,\varphi)$ is indicative surface of the longitudinal piezo-, elasto-optic effect, $\pi'^{(i)}_{in}(\theta,\varphi)$, $p'^{(i)}_{in}(\theta,\varphi)$ and $\pi'^{(m)}_{im}(\theta,\varphi)$ or $p'^{(n)}_{in}(\theta,\varphi)$ are the indicative surfaces of the transverse piezo-, elasto-optic effect for light polarization and mechanical stress or deformation, respectively. Fig.1, Fig.2 (a-c) shows the indicative surfaces for longitudinal (π'_{ii}, p'_{ii} - a) and transverse ($\pi'^{(i)}_{im}$, $p'^{(i)}_{in}$ - b; $\pi'^{(m)}_{im}$, $p'^{(n)}_{in}$- c) piezo-, elasto-optic effect in langasite crystals calculated using the Eqs.(3) - (8) and the data p_{in} presented in the Table 1. More details concerning the method used for a construction of these surfaces is described elsewhere [9, 10].

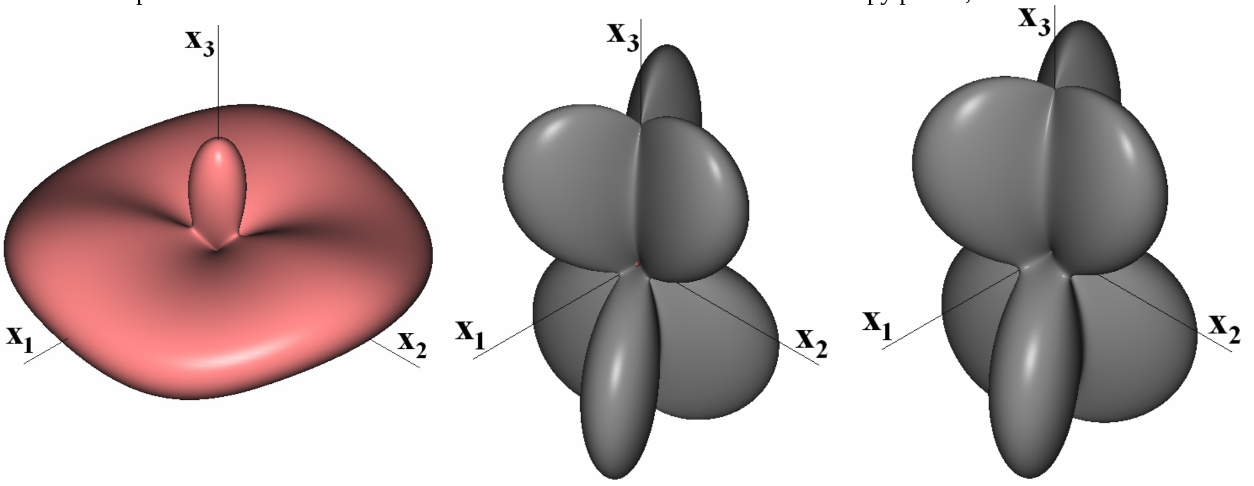
Table 2. The extreme values and anisotropy power for each indicative surface of the piezo- and elasto-optic effect in $La_3Ga_5SiO_{14}$ crystals (see Fig. 1 and Fig. 2)

Indicative surface	Minimal value			Maximal value			Anisotropy power		
	Magnitude	*q*	*j*	Magnitude	*q*	*j*	$V_{sp.}$, (unit)^3	$\|V^+ - V^-\|$, (unit)^3	*h*, %
π'_{ii} (Fig.1a)	-2.05	84	90°, 210° or 330°	-0.32	44	90°, 210° or 330°	36.1	10.6	70.6
$\pi'^{(i)}_{im}$ (Fig.1b)	-0.074	62	30°, 150° or 270°	0.72	28	90°, 210° or 330°	1.56	0.24	85
$\pi'^{(m)}_{im}$ (Fig.1c)	0.023	66	30°, 150° or 270°	0.48	24	90°, 210° or 330°	0.46	0.078	83.2
p'_{ii} (Fig.2a)	-0.17	0°	-	0.053	60°	30°, 150° or 270°	0.021	0.001	95.6
$p'^{(i)}_{in}$ (Fig.2b)	0.026	68°	30°, 150° or 270°	0.091	22°	90°, 210° or 330°	0.003	0.0008	75.5
$p'^{(n)}_{in}$ (Fig.2c)	0.024	63°	30°, 150° or 270°	0.077	27°	90°, 210° or 330°	0.0019	0.0005	73.2

Considering each indicative surface as a function of $f(q,j)$ (i.e. $f(\theta,\varphi) = \pi'_{ii}(\theta,\varphi)$, $p'_{ii}(\theta,\varphi)$, $\pi'^{(i)}_{in}(\theta,\varphi)$, $p'^{(i)}_{in}(\theta,\varphi)$, $\pi'^{(m)}_{im}(\theta,\varphi)$ or $p'^{(n)}_{in}(\theta,\varphi)$) we have calculated the anisotropy power according to the relationship [10]:

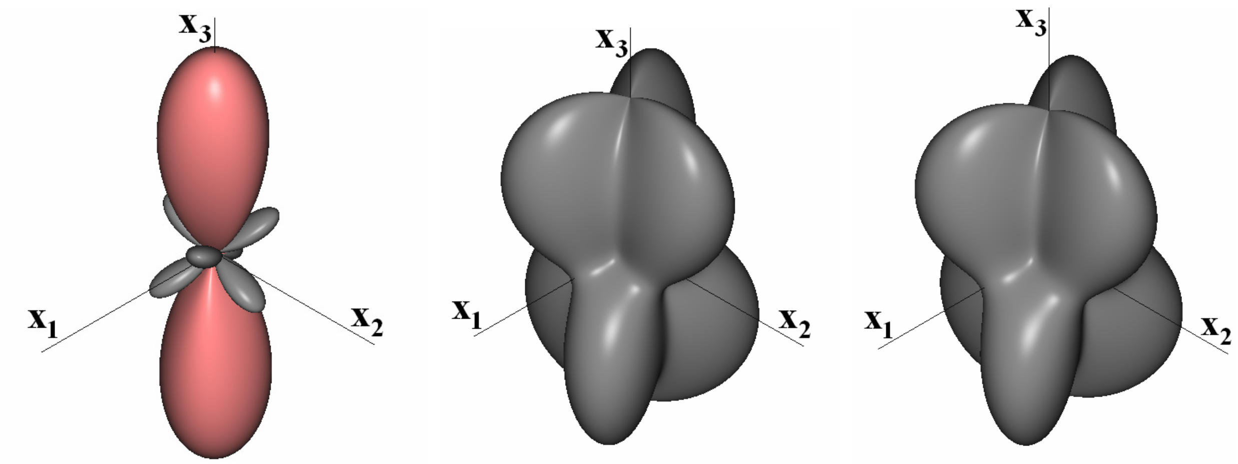
$$h = (V_{sp.} - |V^{+} - V^{-}|)\ 100\% / V_{sp.}\ , \tag{9}$$

where $V_{sp} = 4\pi|f_{extr}|^3/3$ is a sphere volume with radius $|f_{extr}| = \max(|f_{min}|, |f_{max}|)$, V^{+} and V^{-} are the volumes of positive and negative parts of surface, respectively. The anisotropy power defined in such way gives $h = 0\%$ when the indicative surface is a sphere whereas if $V^{+} = V^{-}$ then the effect have the maximum anisotropy power, i.e. $h = 100\%$.



a) b) c)

Fig. 1. The indicative surfaces for longitudinal (π'_{ii} -a) and transverse ($\pi'^{(i)}_{im}$ -b, $\pi'^{(m)}_{im}$ -c) piezo-optic effect of $La_3Ga_5SiO_{14}$ crystals.



a) b) c)

Fig 2. The indicative surfaces for longitudinal (p'_{ii} -a) and transverse ($p'^{(i)}_{in}$ -b, $p'^{(n)}_{in}$ -c) elasto-optical effect of $La_3Ga_5SiO_{14}$ crystals.

3. SPATIAL ANISOTROPY OF THE PIEZO- AND ELASTO-OPTIC EFFECT

Several other features regarding the symmetry follow from the detailed analysis the indicative surfaces in the Fig.1, Fig.2. The surfaces clearly do not have rotation symmetry, what indeed is consistent with the German's theorem [11]. In fact, these surfaces can be described by the point group of symmetry 3m: a threefold axis and three symmetry planes, which are normal to the (X_1, X_2) isotropic plane. This corresponds to well-known Neumann principle [12].

The extreme values and the anisotropy power for each indicative surface of the piezo- and elasto-optic effect in $La_3Ga_5SiO_{14}$ crystals were calculated on the base of our software and represented in Table 2.

4. SPATIAL ANISOTROPY OF THE FIGURE OF MERIT

Analogously, the spatial investigates for various acousto-optic parameters, including effective elasto-optic constants p_{ef}, acoustic wave velocity, acousto-optic figure of merit M_2 were study. The maximal values obtained for the acousto-optic figure of merit M_2 of $La_3Ga_5SiO_{14}$ crystals amount to 72.3×10^{-15} s^3/kg for isotropic and 11×10^{-15} s^3/kg for anisotropic diffraction of light by transverse acoustic waves. The maximal values of light by longitudinal acoustic waves obtained for the acousto-optic figure of merit $M_2 = 28.6\times10^{-15}$ s^3/kg for isotropic diffraction.

Such comparatively large values of the acousto-optic figure of merit are a sound argument concerning the perspective for applying $La_3Ga_5SiO_{14}$ crystals as a good acousto-optic material.

5. CONCLUSIONS

The piezo- and elasto-optic coefficients of $La_3Ga_5SiO_{14}$ crystals were filled for the first time. On the basis of obtained coefficients, the indicative surfaces of piezo-and elasto-optic effects were constructed and their extreme values were determined. The largest value of the figure of merit $M_2 = 72.3\times10^{-15}$ s^3/kg for $La_3Ga_5SiO_{14}$crystals are presented in the case of isotropic diffraction of light by a transverse acoustic wave with lower velocity.

The $La_3Ga_5SiO_{14}$crystals may be considered, therefore, as a candidate for applications in acousto-optical devices.

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